> #Please do not post homework
#Shreya Ghosh, 10-04-2021, Assignment 9

> **read** "/Users/shreyaghosh/Documents/M9.txt"

> *Help9*( )

Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

> #1.
$Orb(2 \cdot x \cdot (1 - x), x, 0.1, 1, 100);$

[0.18, 0.2952, 0.41611392, 0.4859262512, 0.4996038592, 0.4999996862, 0.5000000000] **(2)**

> $SFP(2 \cdot x \cdot (1 - x), x);$

[0.5000000000] **(3)**

> $Orb(2.5 \cdot x \cdot (1 - x), x, 0.1, 1, 100)$

[0.225, 0.4359375, 0.6147399902, 0.5920868365, 0.6038000362, 0.5980638812, 0.6009586880, 0.5995183582, 0.6002402410, 0.5998797352, 0.6000600962, 0.5999699428, 0.6000150262, 0.5999924862, 0.6000037568, 0.5999981215, 0.6000009392, 0.5999995305, 0.6000002348, 0.5999998825, 0.6000000588, 0.5999999705, 0.6000000148, 0.5999999925, 0.6000000038, 0.5999999980, 0.6000000010, 0.5999999995, 0.6000000002, 0.6000000000, **(4)**

0.6000000000, 0.6000000000]

> $SFP(2.5\cdot x\cdot(1-x), x)$

[0.6000000000] **(5)**

> $Orb(3.1\cdot x\cdot(1-x), x, 0.1, 1, 100)$

[0.279, 0.6235929, 0.7276468648, 0.6143484052, 0.7344657708, 0.6045799871, 0.7410953815, 0.5948063527, 0.7471364420, 0.5856630949, 0.7522516860, 0.5777441694, 0.7562631167, 0.5714205665, 0.7591872184, 0.5667481560, 0.7611885195, 0.5635197276, 0.7624922570, 0.5614032265, 0.7633118958, 0.5600672210, 0.7638149799, 0.5592451348, 0.7641190434, 0.5587475058, 0.7643010649, 0.5584493360, 0.7644093928, 0.5582717863, 0.7644736366, 0.5581664560, 0.7645116565, 0.5581041092, 0.7645341288, 0.5580672536, 0.7645474017, 0.5580454841, 0.7645552376, 0.5580326315, 0.7645598625, 0.5580250455, 0.7645625917, 0.5580205688, 0.7645642022, 0.5580179270, 0.7645651523, 0.5580163686, 0.7645657131, 0.5580154488, 0.7645660439, 0.5580149060, 0.7645662392, 0.5580145858, 0.7645663542, 0.5580143970, 0.7645664221, 0.5580142857, 0.7645664624, 0.5580142197, 0.7645664859, 0.5580141810, 0.7645664999, 0.5580141580, 0.7645665082, 0.5580141444, 0.7645665129, 0.5580141369, 0.7645665157, 0.5580141323, 0.7645665176, 0.5580141292, 0.7645665185, 0.5580141276, 0.7645665191, 0.5580141267, 0.7645665194, 0.5580141261, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197] **(6)**

> $SFP(3.1\cdot x\cdot(1-x), x)$

[ ] **(7)**

> $Orb\left(\frac{4+x}{3+x}, x, 0.1, 1, 100\right)$

[1.322580645, 1.231343284, 1.236331570, 1.236053289, 1.236068796, 1.236067932, 1.236067980, 1.236067977, 1.236067978, 1.236067977, **(8)**

```
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
    1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978]
```

> $SFP\left(\frac{4+x}{3+x}, x\right)$

```
                              [1.236067977]                                  (9)
```

> $Orb\left(\frac{3+x}{4+x}, x, 0.1, 1, 100\right)$

```
[0.7560975610, 0.7897435897, 0.7912205568, 0.7912849162, 0.7912877198, 0.7912878419,      (10)
    0.7912878472, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
    0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475]
```

> $SFP\left(\frac{3+x}{4+x}, x\right)$

```
                              [0.791287848]                                  (11)
```

> $Orb\left(\frac{3+x+x^2}{4+x+2\cdot x^2}, x, 0.1, 1, 100\right)$

```
[0.7548543689, 0.7336814751, 0.7352461151, 0.7351314213, 0.7351398339, 0.7351392168,      (12)
    0.7351392622, 0.7351392588, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
    0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
```

0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591]

> $SFP\left(\frac{3+x+x^2}{4+x+2\cdot x^2}, x\right)$

$$[0.7351392587] \qquad \textbf{(13)}$$

>

> #2.

$solve\left(x = \frac{x+a}{x+b}, a\right)$

$$x\,(-1+x+b) \qquad \textbf{(14)}$$

> $solve\left(x = \frac{x+a}{x+b}, b\right)$

$$\frac{-x^2+a+x}{x} \qquad \textbf{(15)}$$

> $diff\left(\frac{x+a}{x+b}, x\right)$

$$\frac{1}{x+b} - \frac{x+a}{(x+b)^2} \qquad \textbf{(16)}$$

> $Orb\left(\frac{x+1}{x+2}, x, 0.1, 1, 100\right)$

[0.5238095238, 0.6037735849, 0.6159420290, 0.6177285319, 0.6179894180, 0.6180274859, **(17)**
0.6180330400, 0.6180338503, 0.6180339685, 0.6180339857, 0.6180339883, 0.6180339886,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,

0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888,
0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888]

> $SFP\left(\frac{x+1}{x+2}, x\right)$

$$[0.6180339880] \tag{18}$$

> $FP\left(\frac{x+1}{x+2}, x\right)$

$$[-1.618033988, 0.6180339880] \tag{19}$$

> $DF(a, b, x) := \frac{1}{x+b} - \frac{x+a}{(x+b)^2}$

$$DF := (a, b, x) \mapsto \frac{1}{x+b} - \frac{x+a}{(x+b)^2} \tag{20}$$

> $DF(1, 2, 0.6180339880)$

$$0.1458980339 \tag{21}$$

> $Orb\left(\frac{x+2}{x+3}, x, 0.1, 1, 100\right)$

[0.6774193548, 0.7280701754, 0.7317647058, 0.7320302648, 0.7320493327, 0.7320507017, **(22)**
0.7320508000, 0.7320508070, 0.7320508075, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,
0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076,

0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076]

> $SFP\left(\frac{x+2}{x+3}, x\right)$

$$[0.732050808] \quad (23)$$

> $FP\left(\frac{x+2}{x+3}, x\right)$

$$[-2.732050808, 0.732050808] \quad (24)$$

> $DF(2, 3, 0.732050808)$

$$0.0717967697 \quad (25)$$

> $SFP\left(\frac{x+12}{x+17}, x\right)$

$$[0.717797888] \quad (26)$$

> $FP\left(\frac{x+12}{x+17}, x\right)$

$$[-16.71779789, 0.717797888] \quad (27)$$

> $DF(12, 17, 0.717797888)$

$$0.01592760650 \quad (28)$$

>

> #3.
#For k=2
$f2(x) := 2 \cdot x \cdot (1 - x);$

$$f2 := x \mapsto 2 \cdot x \cdot (1 - x) \quad (29)$$

> $FP(f2(x), x)$

$$[0., 0.5000000000] \quad (30)$$

> $f2(0.0001)$

$$0.00019998 \quad (31)$$

> $f2(-0.0001)$

$$-0.00020002 \quad (32)$$

> $f2(0.25)$

$$0.3750 \quad (33)$$

> #We can see that x=0 is not a stable fixed point because the system is moving away from the point.

> $f1(x) := x \cdot (1 - x)$

$$f1 := x \mapsto x \cdot (1 - x) \quad (34)$$

> $SFP(f1(x), x)$

$$[\,] \quad (35)$$

> $f3 := 3 \cdot x \cdot (1 - x)$

$$f3 := 3\,x\,(1 - x) \quad (36)$$

> $SFP(f3(x), x)$

Error, (in SFP) cannot determine if this expression is true or false: 3*abs(-(diff(pt(pt), pt))*(1-pt(pt))+pt(pt)*(diff(pt(pt), pt))) < 1

|/Users/shreyaghosh/Documents/M9.txt:55|

> $f4 := 4 \cdot x \cdot (1 - x)$

$$f4 := 4\,x\,(1-x) \quad \textbf{(37)}$$

> $SFP(f4(x), x)$

Error, (in SFP) cannot determine if this expression is true or false: 4*abs(-(diff(pt(pt), pt))*(1-pt(pt))+pt(pt)*(diff(pt(pt), pt))) < 1

|/Users/shreyaghosh/Documents/M9.txt:55|

> *#k=2 is the bifurcation value, with the fixed point being 0.5*

>

> *#4.*

$g(x) := k \cdot x \cdot (1 - x)$

$$g := x \mapsto k \cdot x \cdot (1-x) \quad \textbf{(38)}$$

> $gg := g(g(x))$

$$gg := k^2\,x\,(1-x)\,(1-k\,x\,(1-x)) \quad \textbf{(39)}$$

> $FP(gg, x)$

$$\left[\{k = k, x = 0.\}, \left\{k = k, x = \frac{k-1.}{k}\right\}, \left\{k = k, x = \frac{0.5000000000\,k + 0.5000000000 + 0.5000000000\,\sqrt{k^2 - 2.\,k - 3.}}{k}\right\}, \left\{k = k, x = \frac{0.5000000000\,k + 0.5000000000 - 0.5000000000\,\sqrt{k^2 - 2.\,k - 3.}}{k}\right\}\right] \quad \textbf{(40)}$$

> $dgg := diff(gg, x)$

$$dgg := k^2\,(1-x)\,(1-k\,x\,(1-x)) - k^2\,x\,(1-k\,x\,(1-x)) + k^2\,x\,(1-x)\,(-k\,(1-x) + k\,x) \quad \textbf{(41)}$$

> $solve(\mathrm{abs}(dgg) \le 1)$

Warning, solutions may have been lost

$$\left\{k = 0, x < \frac{1}{2}\right\}, \left\{k = k, x = \frac{1}{2}\right\}, \left\{x = RootOf(27\_Z^4 - 54\_Z^3 + 27\_Z^2 - 2\_Z + 1, index = 1), k \le -27\,RootOf(27\_Z^4 - 54\_Z^3 + 27\_Z^2 - 2\_Z + 1, index = 1)^3 + \frac{81\,RootOf(27\_Z^4 - 54\_Z^3 + 27\_Z^2 - 2\_Z + 1, index = 1)^2}{2} - \frac{27\,RootOf(27\_Z^4 - 54\_Z^3 + 27\_Z^2 - 2\_Z + 1, index = 1)}{2} + 2, \frac{1}{26244}\left(\left(-12\,RootOf(27\_Z^4 - 54\_Z^3 + 27\_Z^2 - 2\_Z + 1, index = 1)^2 + \left(648\,RootOf(27\_Z^4 - 54\_Z^3 + 27\_Z^2 - 2\_Z + 1, index = 1)^2 - 552\,RootOf(27\_Z^4 - 54\_Z^3 + 27\right.\right.\right.$$

$$+ 33\Big)^{1/2} + 12\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 1\right) - 3\Big)$$

$$\Big(53190\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 1\right)^3$$

$$- 68121\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 1\right)^2$$

$$+ 11475\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 1\right) - 5128\Big)\Big) \le k\Big\}, \{x$$

$$= 1, -1 \le k, k \le 1\}, \left\{x = \mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right), k\right.$$

$$\le \frac{1}{26244}\Big(\Big(-12\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right)^2$$

$$+ \Big(648\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right)^2 - 552\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\right.$$

$$+ 33\Big)^{1/2} + 12\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right) - 3\Big)$$

$$\Big(53190\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right)^3$$

$$- 68121\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right)^2$$

$$+ 11475\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right) - 5128\Big)\Big),$$

$$-27\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right)^3$$

$$+ \frac{81\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right)^2}{2}$$

$$\left. - \frac{27\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 - 2\,\_Z + 1, \mathit{index} = 2\right)}{2} + 2 \le k\right\}, \left\{k < 0, x\right.$$

$$< \mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 + 2\,\_Z - 1, \mathit{index} = 3\right), -\frac{1}{2\,(x-1)\,x} < k\Big\}, \Big\{x$$

$$= \mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 + 2\,\_Z - 1, \mathit{index} = 3\right), k < 0,$$

$$27\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 + 2\,\_Z - 1, \mathit{index} = 3\right)^3$$

$$- \frac{81\,\mathit{RootOf}\left(27\,\_Z^4 - 54\,\_Z^3 + 27\,\_Z^2 + 2\,\_Z - 1, \mathit{index} = 3\right)^2}{2}$$

$$+\frac{27\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right)}{2}+2<k\Big\},\Big\{x$$

$$=RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right),k\le-\frac{1}{26244}\Big(\Big($$

$$-12\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right)^2$$

$$+\left(648\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right)^2-744\,RootOf\left(27\_Z^4-54\_Z^3+27\right.\right.$$

$$\left.\left.+129\right)^{1/2}+12\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right)-3\right)$$

$$\left(53190\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right)^3\right.$$

$$-91449\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right)^2$$

$$\left.+34803\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=3\right)+8584\right)\Big),0<k\Big\},\{x$$

$$=0,-1\le k,k<0\},\{x=0,k\le1,0<k\},\Big\{x=RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2\right.$$

$$\left.+2\_Z-1,index=1\right),\frac{1}{26244}\Big(\Big(12\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z\right.$$

$$\left.-1,index=1\right)^2-12\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)$$

$$-\left(648\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)^2-744\,RootOf\left(27\_Z^4-54\_Z^3+27\right.\right.$$

$$\left.\left.+129\right)^{1/2}+3\right)\left(53190\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)^3\right.$$

$$-91449\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)^2$$

$$\left.+34803\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)+8584\right)\Big)\le k,k$$

$$<0\Big\},\Big\{x=RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right),0<k,k$$

$$<27\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)^3$$

$$-\frac{81\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)^2}{2}$$

$$+\frac{27\,RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)}{2}+2\Big\},\Big\{0<k,k<$$

$$-\frac{1}{2\,(x-1)\,x},x<\frac{1}{2},RootOf\left(27\_Z^4-54\_Z^3+27\_Z^2+2\_Z-1,index=1\right)<x\Big\}$$

```
> #I'm sorry Dr. Zeilberger. I'm not sure how to solve this problem
```