#Homework-9
#ok to post
#Richa

read "C:/Users/rmn74/Documents/M9.txt";
Help9( )

Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

#1)
i)
$Orb(2\cdot x\cdot(1-x), x, 0.5, 1000, 1010)$
[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000] **(2)**

$SFP(2\cdot x\cdot(1-x), x)$
[0.5000000000] **(3)**

ii)
$SFP(2.5\cdot x\cdot(1-x), x)$
[0.6000000000] **(4)**

$Orb(2.5\cdot x\cdot(1-x), x, 0.60, 1000, 1010)$
[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000] **(5)**

iii)
$SFP(3.1\cdot x\cdot(1-x), x)$
[ ] **(6)**

$Orb(3.1\cdot x\cdot(1-x), x, 0.9, 1000, 1010)$
[0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197] **(7)**

iv)
$SFP\left(\frac{(4+x)}{3+x}, x\right)$
[1.236067977] **(8)**

$Orb\left(\frac{(4+x)}{3+x}, x, 1.23, 1000, 1010\right)$
[1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978] **(9)**

v)
$SFP\left(\frac{(3+x)}{4+x}, x\right)$
[0.791287848] **(10)**

$Orb\left(\frac{(3+x)}{4+x}, x, 0.79, 1000, 1010\right)$
[0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, **(11)**

0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475]

 # vi)

$$SFP\left(\frac{\left(3+x+x^2\right)}{4+x+2\cdot x^2}, x\right)$$

[0.7351392587] **(12)**

$$Orb\left(\frac{\left(3+x+x^2\right)}{4+x+2\cdot x^2}, x, 0.735, 1000, 1010\right)$$

[0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, **(13)**
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591]

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Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

#2)

$$f(x) := \frac{(x+a)}{x+b}$$

$$f := x \mapsto \frac{x+a}{x+b} \quad \textbf{(2)}$$

solve$(f(x) = x, x)$

$$-\frac{b}{2} + \frac{1}{2} + \frac{\sqrt{b^2+4a-2b+1}}{2}, -\frac{b}{2} + \frac{1}{2} - \frac{\sqrt{b^2+4a-2b+1}}{2} \quad \textbf{(3)}$$

diff$(f(x), x)$

$$\frac{1}{x+b} - \frac{x+a}{(x+b)^2} \quad \textbf{(4)}$$

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Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

#Using a=1 and b=2 to check whether our results match those of the methods

$a := 1$

$$a := 1 \quad \textbf{(2)}$$

$b := 2$

$$b := 2 \quad \textbf{(3)}$$

$$f(x) := \frac{(x+1)}{x+2}$$

$$f := x \mapsto \frac{x+1}{x+2} \quad \textbf{(4)}$$

$FP(f(x), x)$

$$[-1.618033988, 0.6180339880] \quad \textbf{(5)}$$

$SFP(f(x), x)$

$$[0.6180339880] \quad \textbf{(6)}$$

$Orb(f(x), x, 0.6180339880, 1000, 1010)$

$$[0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888] \quad \textbf{(7)}$$

#Using the formula found ahead, we get 2 fixed points similar to those from FP(f,x)

$$evalf\left(\left[-\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4a-2b+1}}{2}, -\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4a-2b+1}}{2}\right]\right)$$

$$[0.6180339880, -1.618033988] \quad \textbf{(8)}$$

$$f'(x) := \frac{1}{x+b} - \frac{(x+a)}{(x+b)^2}$$

$$\frac{d}{dx}\left(\frac{x+1}{x+2}\right) := \frac{1}{x+b} - \frac{x+a}{(x+b)^2} \quad \textbf{(9)}$$

#We check the fixed points whether they are stable or unstable using the formula found ahead

#f'(0.6180229880)is stable as $|f'(x)| < 1$

$f'(0.6180229880)$

$$0.1458992598 \quad \textbf{(10)}$$

#f'(-1.618033988) is unstable as $|f'(x)| > 1$

$f'(-1.618033988)$

$$6.854101940 \quad \textbf{(11)}$$

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Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

#Using a=3 and b=12 to check whether our results match those of the methods

$a := 3$

$$a := 3 \quad \textbf{(2)}$$

$b := 12$

$$b := 12 \quad \textbf{(3)}$$

$$f(x) := \frac{(x+3)}{x+12}$$

$$f := x \mapsto \frac{3+x}{x+12} \quad \textbf{(4)}$$

$FP(f(x), x)$

$$[-11.26628130, 0.266281295] \quad \textbf{(5)}$$

$SFP(f(x), x)$

$$[0.266281295] \quad \textbf{(6)}$$

$Orb(f(x), x, 0.266281295, 1000, 1010)$

$$[0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973, 0.2662812973] \quad \textbf{(7)}$$

#Using the formula found ahead, we get 2 fixed points similar to those from FP(f,x)

$$evalf\left(\left[-\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4a-2b+1}}{2}, -\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4a-2b+1}}{2}\right]\right)$$

$$[0.266281295, -11.26628130] \quad \textbf{(8)}$$

$$f'(x) := \frac{1}{x+b} - \frac{(x+a)}{(x+b)^2}$$

$$\frac{d}{dx}\left(\frac{3+x}{x+12}\right) := \frac{1}{x+b} - \frac{x+a}{(x+b)^2} \quad \textbf{(9)}$$

#We check the fixed points whether they are stable or unstable using the formula found ahead

#f'(0.266281295)is stable as $|f'(x)| < 1$

$f'(0.266281295)$

$$0.05981590385 \quad \textbf{(10)}$$

#` f'(-11.26628130) is unstable as $|f'(x)| > 1$

$f'(-11.26628130)$

$$16.71796200 \quad \textbf{(11)}$$

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Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

#Using a=12 and b=17 to check whether our results match those of the methods

$a := 12$

$$a := 12 \quad \textbf{(2)}$$

$b := 17$

$$b := 17 \quad \textbf{(3)}$$

$f(x) := \frac{(x+12)}{x+17}$

$$f := x \mapsto \frac{x+12}{x+17} \quad \textbf{(4)}$$

$FP(f(x), x)$

$$[-16.71779789, 0.717797888] \quad \textbf{(5)}$$

$SFP(f(x), x)$

$$[0.717797888] \quad \textbf{(6)}$$

$Orb(f(x), x, 0.717797888, 1000, 1010)$

$$[0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871] \quad \textbf{(7)}$$

#Using the formula found ahead, we get 2 fixed points similar to those from FP(f,x)

$evalf\left(\left[-\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4a-2b+1}}{2}, -\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4a-2b+1}}{2}\right]\right)$

$$[0.717797888, -16.71779789] \quad \textbf{(8)}$$

$f'(x) := \frac{1}{x+b} - \frac{(x+a)}{(x+b)^2}$

$$\frac{d}{dx}\left(\frac{x+12}{x+17}\right) := \frac{1}{x+b} - \frac{x+a}{(x+b)^2} \quad \textbf{(9)}$$

#We check the fixed points whether they are stable or unstable using the formula found ahead

#f'(0.717797888)is stable as $|f'(x)| < 1$

$f'(0.717797888)$

$$0.01592760650 \quad \textbf{(10)}$$

#` f'(-16.71779789) is unstable as $|f'(x)| > 1$

$f'(-16.71779789)$

$$62.78407369 \quad \textbf{(11)}$$

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Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x) **(1)**

#3)

$$f(x) := k \cdot x \cdot (1 - x)$$

$$f := x \mapsto k \cdot x \cdot (1 - x) \quad \textbf{(2)}$$

```
for i from 1 to 4 do
 k := i :
 print(f(x));
 m := [solve(f(x) = x, x)];
 print(m);
 print(f'(x));
 print(f'(m[1]));
 print(f'(m[2]));
 print( );
 od:
```

$$x\,(1 - x)$$

$$[0, 0]$$

$$1 - 2\,x$$

$$1$$

$$1$$

$$2\,x\,(1 - x)$$

$$\left[0, \frac{1}{2}\right]$$

$$2 - 4\,x$$

$$2$$

$$0$$

$$3\,x\,(1 - x)$$

$$\left[0, \frac{2}{3}\right]$$

$$3 - 6\,x$$

$$3$$

$$-1$$

$$4\,x\,(1 - x)$$

$$\left[0, \frac{3}{4}\right]$$

$$4 - 8\,x$$

$$4$$

$$-2$$

(3)

 # The second fixed point is stable for 2·x·(1-x), and the first bifurcation value is at that point as well

#4)

$F(x) := f(f(x))$

$$F := x \mapsto f(f(x))$$ **(4)**

for i **from** 1 **to** 4 **do**

$k := i$:

$print(F(x))$;

$m := [solve(F(x) = x, x)]$;

$print(m)$;

$print(F'(x))$;

$print(F'(m[1]))$;

$print(F'(m[2]))$;

$print(Orb(F(x), x, 0.5, 1000, 1010))$;

$print()$;

od:

$$x\,(1-x)\,(1-x\,(1-x))$$

$$[0, 0, 1 + I, 1 - I]$$

$$(1-x)\,(1-x\,(1-x)) - x\,(1-x\,(1-x)) + x\,(1-x)\,(-1+2\,x)$$

$$1$$

$$1$$

$$[0.0004976675995, 0.0004971724999, 0.0004966783846, 0.0004961852508,$$
$$0.0004956930954, 0.0004952019157, 0.0004947117087, 0.0004942224714,$$
$$0.0004937342010, 0.0004932468947, 0.0004927605496, 0.0004922751630]$$

$$4\,x\,(1-x)\,(1-2\,x\,(1-x))$$

$$\left[0, \frac{1}{2}, \frac{3}{4} - \frac{I\sqrt{3}}{4}, \frac{3}{4} + \frac{I\sqrt{3}}{4}\right]$$

$$4\,(1-x)\,(1-2\,x\,(1-x)) - 4\,x\,(1-2\,x\,(1-x)) + 4\,x\,(1-x)\,(-2+4\,x)$$

$$4$$

$$0$$

$$[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,$$
$$0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000]$$

$$9\,x\,(1-x)\,(1-3\,x\,(1-x))$$

$$\left[0, \frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right]$$

$$9\,(1-x)\,(1-3\,x\,(1-x)) - 9\,x\,(1-3\,x\,(1-x)) + 9\,x\,(1-x)\,(-3+6\,x)$$

$$9$$

$$1$$

$$[0.6613716248, 0.6613742758, 0.6613769229, 0.6613795659, 0.6613822051, 0.6613848402, 0.6613874714, 0.6613900987, 0.6613927223, 0.6613953418, 0.6613979576, 0.6614005694]$$

$$16\,x\,(1-x)\,(1-4\,x\,(1-x))$$

$$\left[0, \frac{3}{4}, \frac{5}{8} - \frac{\sqrt{5}}{8}, \frac{5}{8} + \frac{\sqrt{5}}{8}\right]$$

$$16\,(1-x)\,(1-4\,x\,(1-x)) - 16\,x\,(1-4\,x\,(1-x)) + 16\,x\,(1-x)\,(8\,x-4)$$

$$16$$

$$4$$

$$[0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.]$$

(5)

#Here we directly know from our previous calculations that fixed points are not stable for when k=1,3 as they are same as our previous que. We also know that for k=2 we get |F'(x)|<1, therefore it is stable. For k=4, it is stable at F'(0.5), we are able to check this using Orb function and this is the second bifurcation point.