```
> #Do not post
 #Nikita John, Assigment 9
 #October 4th, 2021
> #M9.txt: Maple Code for "Dynamical models in Biology" (Math 336) taught by Dr. Z., Lecture 9

Help9 :=proc( ) :
    print( ` Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)  `) :end:


    #Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point,
    x0, and a positive integer K, outputs
#the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
#Orb(2*x*(1-x),x,0.4,1000,2000);
Orb :=proc( f, x, x0, K1, K2 ) local x1, i, L :
x1 := x0 :
for i from 1 to K1 do
 x1 := subs(x=x1, f ) :
    #we don't record the first values of K1, since we are interested in the long-time behavior of
    the orbit
od:

L := [x1] :

for i from K1 to K2 do
 x1 := subs(x=x1, f ) :   #we compute the next member of the orbit
 L := [op(L), x1] : #we append it to the list
od:

L : #that's the output

end:

#Orb2D(f,x,x0,K): 2D version of Orb(f,x,x0,0,K), just for illustration
Orb2D :=proc( f, x, x0, K ) local L, L1, i :
L := Orb( f, x, x0, 0, K ) :
L1 := [ [L[1], 0], [L[1], L[2]], [L[2], L[2]] ] :
for i from 3 to nops(L) do
 L1 := [op(L1), [L[i-1], L[i]], [L[i], L[i]]] :
od:
L1 :
end:


#FP(f,x): The list of fixed points of the map x->f where f is an expression in x. Try:
#FP(2*x*(1-x),x);
FP :=proc( f, x )
evalf ( [solve( f=x ) ] ) :
end:
```

```
#SFP(f,x): The list of stable fixed points of the map x->f where f is an expression in x. Try:
#SFP(2*x*(1-x),x);
SFP :=proc(f, x) local L, i, f1, pt, Ls :
L := FP(f, x) : #The list of fixed points (including complex ones)

Ls := [ ] :       #Ls is the list of stable fixed points, that starts out as the empty list

f1 := diff(f, x) :  #The derivative of the function f w.r.t. x

for i from 1 to nops(L) do
pt := L[i] :

if abs(subs(x=pt, f1)) < 1 then

 Ls := [op(Ls), pt] :  # if pt,  is stable we add it to the list of stable points

fi:

od:

Ls :   #The last line is the output

end:

#Comp(f,x): f(f(x))
Comp :=proc(f, x) : normal(subs(x=f, f) ) :end:
```

```
> #1(i)
  f1 := 2·x·(1 − x) :
  Orb(f1, x, 0.5, 990, 1000);
  SFP(f1, x);
```

[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000]

[0.5000000000] **(1)**

```
> #1(ii)
  f2 := 2.5·x·(1 − x) :
  Orb(f2, x, 0.5, 990, 1000);
  SFP(f2, x);
```

[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000]

[0.6000000000] **(2)**

```
> #1(iii)
  f3 := 3.1·x·(1 − x) :
  Orb(f3, x, 0.5, 990, 1000);
```

$SFP(f3, x);$

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203]

$$[\ ] \tag{3}$$

\> *#1(iv)*

$$f4 := \frac{(4+x)}{(3+x)}:$$

$Orb(f4, x, 0.5, 990, 1000);$

$SFP(f4, x);$

[1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978]

$$[1.236067977] \tag{4}$$

\> *#1(v)*

$$f5 := \frac{(3+x)}{(4+x)}:$$

$Orb(f5, x, 0.5, 990, 1000);$

$SFP(f5, x);$

[0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475]

$$[0.791287848] \tag{5}$$

\> *#1(vi)*

$$f6 := \frac{(3+x+x^2)}{(4+x+2\cdot x^2)}:$$

$Orb(f6, x, 0.5, 990, 1000);$

$SFP(f6, x);$

[0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591]

$$[0.7351392587] \tag{6}$$

\> $solve\left(\frac{(x+a)}{(x+b)}, x\right);$

$$-a \tag{7}$$

\> $diff(\%, x);$

$$0 \tag{8}$$

\> *#a = 1, b = 2*

$$F1 := \frac{(x+1)}{(x+2)}:$$

$solve(F1, x);$

$$-1 \tag{9}$$

> *diff*(%, *x*);

$$0 \quad \textbf{(10)}$$

> *Orb*(*F1*, *x*, 0.5, 990, 1000);
FP(*F1*, *x*);
SFP(*F1*, *x*);

[0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888]

$$[-1.618033988, 0.6180339880]$$

$$[0.6180339880] \quad \textbf{(11)}$$

> *#a = 2, b =3*

$$F2 := \frac{(x+2)}{(x+3)} :$$

solve(*F2*, *x*);

$$-2 \quad \textbf{(12)}$$

> *diff*(%, *x*);

$$0 \quad \textbf{(13)}$$

> *Orb*(*F2*, *x*, 0.5, 990, 1000);
FP(*F2*, *x*);
SFP(*F2*, *x*);

[0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076]

$$[-2.732050808, 0.732050808]$$

$$[0.732050808] \quad \textbf{(14)}$$

> *#a = 12, b = 17*

$$F3 := \frac{(x+12)}{(x+17)} :$$

solve(*F3*, *x*);

$$-12 \quad \textbf{(15)}$$

> *diff*(%, *x*);

$$0 \quad \textbf{(16)}$$

> *Orb*(*F3*, *x*, 0.5, 990, 1000);
FP(*F3*, *x*);
SFP(*F3*, *x*);

[0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871]

$$[-16.71779789, 0.717797888]$$

$$[0.717797888] \quad \textbf{(17)}$$

> *#3: I did various test equations with different values of k, and found the fixed points and stable fixed points. Each time, the stable fixed point usually consists of the non-zero value*

$P0 := k \cdot x \cdot (1 - x) - x :$
$solve(P0, x);$
$Pdiff := diff(P0, x);$
$subs(x = 0, Pdiff);$
$subs\left(x = \frac{(k-1)}{k}, Pdiff\right);$

#when we substitute 0 in for x, we get k -1 , which if k is between 1 and 4, will always be greater than 1, which is unstable. The second point can be less than one if k is less than 3, meaning the point is sometimes stable

$$0, \frac{k-1}{k}$$
$$Pdiff := k(1 - x) - kx - 1$$
$$k - 1$$
$$k\left(1 - \frac{k-1}{k}\right) - k \quad \textbf{(18)}$$

> $P1 := 2 \cdot x \cdot (1 - x) :$
$FP(P1, x);$
$SFP(P1, x);$

$$[0., 0.5000000000]$$
$$[0.5000000000] \quad \textbf{(19)}$$

> $P2 := 2.5 \cdot x \cdot (1 - x) :$
$FP(P2, x);$
$SFP(P2, x);$

$$[0., 0.6000000000]$$
$$[0.6000000000] \quad \textbf{(20)}$$

> $P3 := 3.2 \cdot x \cdot (1 - x) :$
$FP(P3, x);$
$SFP(P3, x);$

$$[0., 0.6875000000]$$
$$[\ ] \quad \textbf{(21)}$$

> $P4 := 2.75 \cdot x \cdot (1 - x) :$
$FP(P4, x);$
$SFP(P4, x);$

$$[0., 0.6363636364]$$
$$[0.6363636364] \quad \textbf{(22)}$$

> $P5 := 3 \cdot x \cdot (1 - x);$
$FP(P5, x);$
$SFP(P5, x);$

#For k <3, the other point is stable. Once k = 3, then neither of the points are stable , and we hit the first bifurcation point.

$$P5 := 3x(1 - x)$$
$$[0., 0.6666666667]$$

$$[\ ]$$ **(23)**

```
> #4
P0_r := k·x·(1 − x) :
PComp := Comp(P0_r, x);
Psolve := PComp − x :
solve(Psolve, x);
PCompSDiff := diff(Psolve, x);
```

$$PComp := -k^2 x\,(-1+x)\,\left(k\,x^2 - k\,x + 1\right)$$

$$0,\ \frac{k-1}{k},\ \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2\,k-3}}{2}}{k},\ \frac{\frac{k}{2}+\frac{1}{2}-\frac{\sqrt{k^2-2\,k-3}}{2}}{k}$$

$$PCompSDiff := -k^2\,(-1+x)\,\left(k\,x^2-k\,x+1\right) - k^2\,x\,\left(k\,x^2-k\,x+1\right) - k^2\,x\,(-1+x)\,(2\,k\,x - k) - 1$$ **(24)**

```
> P1_r := 3.75·x·(1 − x);
P1Comp := Comp(P1_r, x);
```

$$P1\_r := 3.75\,x\,(1-x)$$

$$P1Comp := -14.0625\,x\,(-1+x)\,\left(1+3.75\,x^2-3.75\,x\right)$$ **(25)**

```
> # I wasn't sure how exactly to find at what value of k we reach the second bifurcation point, but I
    surmised that the value of k would be between 3 and 4, since the first bifurcation point
    occured when k was 3. So I tried various values and got that it occured at 3.75.
Orb(P1Comp, x, 0.5, 990, 1000);
FP(P1Comp, x);
SFP(P1Comp, x);
```

[0., 0.7333333333, 0.3816721855, 0.8849944812]

$$[\ ]$$ **(26)**

```
>
```