#OK to post
#Julian Herman, 10/4/21, Assignment 9

3) Find equilibrium points:

$x = Kx(1-x)$

$x - Kx + Kx^2 = 0$

$x(1 - K + Kx) = 0$

$x(1 - K(1-x)) = 0$

$x = 0 \qquad 1 - K(1-x) = 0$

$1 = K(1-x)$

$\frac{1}{K} = 1 - x$

$$\boxed{x = 0, \quad x = 1 - \tfrac{1}{K} = \frac{K-1}{K}}$$

For $x=0$ to be stable, $|f'(x=0)| < 1$

$\hookrightarrow f(x) = Kx(1-x)$

$f(x) = Kx - Kx^2$

$f'(x) = K - 2Kx$

$f'(0) = K - \cancel{2K \cdot 0} = K$

$|K| < 1$ for $x=0$ to be stable

$\Rightarrow$ Since K must be between 1 and 4, $x=0$ will never be stable because:

$|K| < 1$ is not true for any $1 \leq K \leq 4$.

Check stability of $x = \frac{K-1}{K}$:

$$f'\left(x = \frac{K-1}{K}\right) = K - 2K\left(\frac{K-1}{K}\right) = K - 2(K-1) = 2-K$$

$|2-K| < 1$ is when $x = \frac{K-1}{K}$ is stable

$-1 < 2-K$, $2-K < 1$

$K < 3$, $1 < K$

$\Rightarrow$ $x = \frac{K-1}{K}$ is stable for $1 < K < 3$

$\Rightarrow$ Therefore, the first bifurcation point occurs at $K = 3$, when the fixed point is no longer stable and the population oscillates between more than one value (in the $K=3$ case, it fluctuates between 2 values).