```
> #Question 1
> #Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point, x0,
      and a positive integer K, outputs
  #the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
  #Orb(2*x*(1-x),x,0.4,1000,2000);
  Orb := proc(f, x, x0, K1, K2) local x1, i, L :
  x1 := x0 :
  for i from 1 to K1 do
   x1 := subs(x = x1, f) :
      #we don't record the first values of K1, since we are interested in the long-time behavior of
      the orbit
  od:

  L := [x1] :

  for i from K1 to K2 do
   x1 := subs(x = x1, f) :   #we compute the next member of the orbit
   L := [op(L), x1] : #we append it to the list
  od:

  L : #that's the output

  end:
> #FP(f,x): The list of fixed points of the map x->f where f is an expression in x. Try:
  #FP(2*x*(1-x),x);
  FP := proc(f, x)
  evalf([solve(f = x)]) :
  end:


  #SFP(f,x): The list of stable fixed points of the map x->f where f is an expression in x. Try:
  #SFP(2*x*(1-x),x);
  SFP := proc(f, x) local L, i, f1, pt, Ls :
  L := FP(f, x) : #The list of fixed points (including complex ones)

  Ls := [ ] :       #Ls is the list of stable fixed points, that starts out as the empty list

  f1 := diff(f, x) :  #The derivative of the function f w.r.t. x

  for i from 1 to nops(L) do
  pt := L[i] :

  if abs(subs(x = pt, f1)) < 1 then

   Ls := [op(Ls), pt] :  # if pt,  is stable we add it to the list of stable points

  fi:

  od:
```

```
    Ls :   #The last line is the output

    end:
> # (i)
> SFP(2·x·(1 − x), x);
                                  [0.5000000000]                                  (1)
> Orb(2·x·(1 − x), x, .1, 1, 50);
[0.18, 0.2952, 0.41611392, 0.4859262512, 0.4996038592, 0.4999996862, 0.5000000000,   (2)
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000]
> # SFP at 0.50
> #(ii)
> SFP(2.5·x·(1 − x), x);
                                  [0.6000000000]                                  (3)
> Orb(2.5·x·(1 − x), x, .1, 1, 50);
[0.225, 0.4359375, 0.6147399902, 0.5920868365, 0.6038000362, 0.5980638812,       (4)
    0.6009586880, 0.5995183582, 0.6002402410, 0.5998797352, 0.6000600962,
    0.5999699428, 0.6000150262, 0.5999924862, 0.6000037568, 0.5999981215,
    0.6000009392, 0.5999995305, 0.6000002348, 0.5999998825, 0.6000000588,
    0.5999999705, 0.6000000148, 0.5999999925, 0.6000000038, 0.5999999980,
    0.6000000010, 0.5999999995, 0.6000000002, 0.6000000000, 0.6000000000,
    0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
    0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
    0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
    0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000]
> # SFP at 0.60
> #(iii)
> SFP(3.1·x·(1 − x), x);
                                        [ ]                                       (5)
> Orb(3.1·x·(1 − x), x, .1, 1, 50);
[0.279, 0.6235929, 0.7276468648, 0.6143484052, 0.7344657708, 0.6045799871,       (6)
    0.7410953815, 0.5948063527, 0.7471364420, 0.5856630949, 0.7522516860,
    0.5777441694, 0.7562631167, 0.5714205665, 0.7591872184, 0.5667481560,
```

0.7611885195, 0.5635197276, 0.7624922570, 0.5614032265, 0.7633118958,
0.5600672210, 0.7638149799, 0.5592451348, 0.7641190434, 0.5587475058,
0.7643010649, 0.5584493360, 0.7644093928, 0.5582717863, 0.7644736366,
0.5581664560, 0.7645116565, 0.5581041092, 0.7645341288, 0.5580672536,
0.7645474017, 0.5580454841, 0.7645552376, 0.5580326315, 0.7645598625,
0.5580250455, 0.7645625917, 0.5580205688, 0.7645642022, 0.5580179270,
0.7645651523, 0.5580163686, 0.7645657131, 0.5580154488, 0.7645660439]

> *# No stable fixed points*

> *#(iv)*

> $SFP\left(\frac{(4+x)}{(3+x)}, x\right);$

[1.236067977] **(7)**

> $Orb\left(\frac{(4+x)}{(3+x)}, x, .1, 1, 50\right);$

[1.322580645, 1.231343284, 1.236331570, 1.236053289, 1.236068796, 1.236067932, **(8)**
1.236067980, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977,
1.236067978, 1.236067977, 1.236067978]

> *# Stable fixed point at 1.236*

> *#(v)*

> $SFP\left(\frac{(3+x)}{(4+x)}, x\right);$

[0.791287848] **(9)**

> $Orb\left(\frac{(3+x)}{(4+x)}, x, .1, 1, 50\right);$

[0.7560975610, 0.7897435897, 0.7912205568, 0.7912849162, 0.7912877198, 0.7912878419, **(10)**
0.7912878472, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,
0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475,

0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475]

```
> # stable fixed point at .791
> #(vi)
> SFP((3 + x + x^2)/(4 + x + 2·x^2), x);
```

[0.7351392587] (11)

```
> Orb((3 + x + x^2)/(4 + x + 2·x^2), x, .1, 1, 50);
```

[0.7548543689, 0.7336814751, 0.7352461151, 0.7351314213, 0.7351398339, 0.7351392168, (12)
0.7351392622, 0.7351392588, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591,
0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591]

```
> # Stable fixed point at .735
> #Question 2
> help(solve);
> solve(x - (x + a)/(x + b) = 0, x);
```

$$-\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4a-2b+1}}{2},\ -\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4a-2b+1}}{2} \quad (13)$$

```
> help(diff);
> diff((x + a)/(x + b), x);
```

$$\frac{1}{x+b}-\frac{x+a}{(x+b)^2} \quad (14)$$

```
> # a must be less than b+b^2 and greater than  b - b^2 if |C(a, b)| < 1
> FP((x + 1)/(x + 2), x);
```

[−1.618033988, 0.6180339880] (15)

```
> SFP((x + 1)/(x + 2), x);
```

[0.6180339880] (16)

```
> evalf(Orb((x + 1)/(x + 2), x, .618, 990, 1000));
```

$$[0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888] \quad (17)$$

> $evalf\left(Orb\left(\frac{(x+1)}{(x+2)}, x, .718, 990, 1000\right)\right);$

$$[0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888] \quad (18)$$

> $evalf\left(Orb\left(\frac{(x+1)}{(x+2)}, x, .518, 990, 1000\right)\right);$

$$[0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888] \quad (19)$$

> $FP\left(\frac{(x+2)}{(x+3)}, x\right);$

$$[-2.732050808, 0.732050808] \quad (20)$$

> $SFP\left(\frac{(x+2)}{(x+3)}, x\right);$

$$[0.732050808] \quad (21)$$

> $evalf\left(Orb\left(\frac{(x+2)}{(x+3)}, x, .732, 990, 1000\right)\right);$

$$[0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076] \quad (22)$$

> $evalf\left(Orb\left(\frac{(x+2)}{(x+3)}, x, .832, 990, 1000\right)\right);$

$$[0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076] \quad (23)$$

> $evalf\left(Orb\left(\frac{(x+2)}{(x+3)}, x, .632, 990, 1000\right)\right);$

$$[0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076] \quad (24)$$

> $FP\left(\frac{(x+12)}{(x+17)}, x\right);$

$$[-16.71779789, 0.717797888] \quad (25)$$

> $SFP\left(\frac{(x+12)}{(x+17)}, x\right);$

$$[0.717797888] \quad (26)$$

```
> evalf(Orb((x + 12)/(x + 17), x, .718, 990, 1000));
```

[0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871] **(27)**

```
> evalf(Orb((x + 12)/(x + 17), x, .818, 990, 1000));
```

[0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871] **(28)**

```
> evalf(Orb((x + 12)/(x + 17), x, .618, 990, 1000));
```

[0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871] **(29)**

```
> #Question 3
> FP(x·(1 − x), x);
```

[0., 0.] **(30)**

```
> SFP(x·(1 − x), x);
```

[ ] **(31)**

```
> FP(2·x·(1 − x), x);
```

[0., 0.5000000000] **(32)**

```
> SFP(2·x·(1 − x), x);
```

[0.5000000000] **(33)**

```
> FP(3·x·(1 − x), x);
```

[0., 0.6666666667] **(34)**

```
> SFP(3·x·(1 − x), x);
```

[ ] **(35)**

```
> FP(4·x·(1 − x), x);
```

[0., 0.7500000000] **(36)**

```
> SFP(4·x·(1 − x), x);
```

[ ] **(37)**

```
> #At k=2, the other fixed point is stable. At no value of k is (0,0) stable
> Orb(2.6*x·(1 − x), x, .1, 990, 1000);
```

[0.6153846153, 0.6153846156, 0.6153846153, 0.6153846156, 0.6153846153, 0.6153846156, 0.6153846153, 0.6153846156, 0.6153846153, 0.6153846156, 0.6153846153, 0.6153846156] **(38)**

```
> #At k=2.6, the population switches from being constant at a single value to oscillating between
    two values (bifurcation)
> #Question 4
```

```
> #Comp(f,x): f(f(x))
  Comp := proc(f, x) : normal(subs(x = f, f) ) :end:
> Comp( 1.4142· x·(1 − x), x);
```

$$-1.99996164\,x\,(-1+x)\left(1+1.4142\,x^2-1.4142\,x\right) \tag{39}$$

```
> solve(x − %, x);
```

$$0.,\ 0.2928864376,\ 0.8535567812-0.6917831841\,I,\ 0.8535567812+0.6917831841\,I \tag{40}$$

```
> SFP(x − %%, x);
```

$$[0.] \tag{41}$$

```
> FP(%%%, x);
```

$$[0.,\ 0.2928864376,\ 0.8535567812-0.6917831841\,I,\ 0.8535567812+0.6917831841\,I] \tag{42}$$

```
>
```