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> #Not OK to post
> #Anusha Nagar, Homework 9, 10.4.2021
>
> read "C:/Users/an646/Documents/M9.txt"
> Help9( )
            Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)      (1)
> #Problem 1
> #1a
> f1 := 2·x·(1 − x);
                              f1 := 2 x (1 − x)                                (2)
> Orb(f1, x, 0.9, 1000, 1020)
[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,   (3)
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
    0.5000000000]
> SFP(f1, x)
                              [0.5000000000]                                   (4)
> #Stable fixed point at 0.5
>
> #1b
> f2 := 2.5·x·(1 − x);
                              f2 := 2.5 x (1 − x)                              (5)
> Orb(f2, x, 0.9, 1000, 1020)
[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,   (6)
    0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
    0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
    0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
    0.6000000000]
> SFP(f2, x)
                              [0.6000000000]                                   (7)
> #Stable fixed point at 0.6
>
> #1c
> f3 := 3.1·x·(1 − x);
                              f3 := 3.1 x (1 − x)                              (8)
> Orb(f3, x, 0.6, 1000, 1020)
[0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197,   (9)
    0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258,
    0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197,
    0.5580141258, 0.7645665197, 0.5580141258, 0.7645665197, 0.5580141258,
    0.7645665197]
> SFP(f3, x)
                              [ ]                                              (10)
```

> *#SFP fluctuates between 0.558 and 0.7646*

>

> *#1d*

> $f4 := \frac{(4+x)}{(3+x)};$

$$f4 := \frac{4+x}{3+x} \quad (11)$$

> $Orb(f4, x, 0.9, 1000, 1020)$

[1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978] **(12)**

> $SFP(f4, x)$

[1.236067977] **(13)**

> *#Stable fixed point at 1.236*

>

> *#1e*

> $f5 := \frac{(3+x)}{(4+x)};$

$$f5 := \frac{3+x}{4+x} \quad (14)$$

> $Orb(f5, x, 0.9, 1000, 1020)$

[0.7912878475, 0.7912878475] **(15)**

> $SFP(f5, x)$

[0.791287848] **(16)**

> *#Stable fixed point at 0.7913*

>

> *#1f*

> $f6 := \frac{(3+x+x^2)}{(4+x+2\cdot x^2)};$

$$f6 := \frac{x^2+x+3}{2x^2+x+4} \quad (17)$$

> $Orb(f6, x, 0.9, 1000, 1020)$

[0.7351392591, 0.7351392591] **(18)**

> $SFP(f6, x)$

(19)

$$[0.7351392587] \quad (19)$$

> #stable fixed point at 0.735

>

> #Problem 2

> #x = (x+a) over (x+b)

> $\#x^2 + bx - x - a = 0$

> $solve\left(x^2 + b \cdot x - x - a, x\right)$

$$-\frac{b}{2} + \frac{1}{2} + \frac{\sqrt{b^2 + 4a - 2b + 1}}{2}, -\frac{b}{2} + \frac{1}{2} - \frac{\sqrt{b^2 + 4a - 2b + 1}}{2} \quad (20)$$

> #Above we have our values for equilibrium points

> $f\_prime\_x := 2x + b$

$$f\_prime\_x := 2x + b \quad (21)$$

> $x\_2\_1 := -\frac{b}{2} + \frac{1}{2} + \frac{\sqrt{b^2 + 4a - 2b + 1}}{2};$

$$x\_2\_1 := -\frac{b}{2} + \frac{1}{2} + \frac{\sqrt{b^2 + 4a - 2b + 1}}{2} \quad (22)$$

> $x\_2\_2 := -\frac{b}{2} + \frac{1}{2} - \frac{\sqrt{b^2 + 4a - 2b + 1}}{2};$

$$x\_2\_2 := -\frac{b}{2} + \frac{1}{2} - \frac{\sqrt{b^2 + 4a - 2b + 1}}{2} \quad (23)$$

> $f\_prime\_2\_1 := 2 \cdot x\_2\_1 + b;$

$$f\_prime\_2\_1 := 1 + \sqrt{b^2 + 4a - 2b + 1} \quad (24)$$

> $f\_prime\_2\_2 := 2 \cdot x\_2\_2 + b$

$$f\_prime\_2\_2 := 1 - \sqrt{b^2 + 4a - 2b + 1} \quad (25)$$

> #f prime must be < 1 **for** stable fixed points : here we see that this must be $C(a, b) = 1 + - \mathrm{sqrt} b^2 - 4a - 2b + 1$ between -1 **and** 1 **for** stable fixed points

>

> #C(1,1) = 1+-(2) is between -1 and 1. Should have stable fixed points. C(2,3)= not between -1 and 1 (-2.46 and 4.46). Should have no SFP. C(12,17) not between -1 and 1 (-16.42 and 18.43). Should have no SFP

> $SFP\left(\frac{(x+1)}{(x+1)}, x\right)$

$$[1.] \quad (26)$$

> $SFP\left(\frac{(x+2)}{(x+3)}, x\right)$

$$[0.732050808] \quad (27)$$

> $SFP\left(\frac{(x+12)}{(x+17)}, x\right)$

$$[0.717797888] \quad (28)$$

> #Problem 3

> #x=kx(1-x)

> #x = kx-kx^2

> #0=x[k-kx-1]

> #x = 0 or x=(k-1)divided by k

> #f'x = k-2kx

> #When x = 0, f'(x) = k -> this is not stable for k between 1-4, as stable fixed points exist only when f'(x)<1

> #Second equil point at (k-1)·k^{-1}

> #Here, f'(x)=k-2·(k-1) = 2-k

> #For this point to be a SFP, we need k>1.

> #When k =3, f(x) = 3x-3x^2 $\rightarrow$ equilibrium points at $x = 0$ **and** $x = \frac{2}{3}$

> #$\frac{\text{For } x=2}{3}$, $f'(x) = -1$ - > first bifurcation

>

> #Problem 4

> #Here, we are doing x->f(f(x)) or x-> f(kx(1-x))

> #This is basically x->k[kx(1-x)](1-[kx(1-x)])

> #Next, we set x = k[kx(1-x)](1-[kx(1-x)])

>

> $solve\left(x = \left(k^2 \cdot x - k^2 \cdot x^2\right) \cdot \left(1 - k \cdot x + k \cdot x^2\right)\right)$

$$\{k=k, x=0\}, \left\{k=k, x=\frac{k-1}{k}\right\}, \left\{k=k, x=\frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}\right\}, \left\{k=k, x=\frac{\frac{k}{2}+\frac{1}{2}-\frac{\sqrt{k^2-2k-3}}{2}}{k}\right\} \quad \textbf{(29)}$$

>

> #Here, we see our same fixed points as earlier (x=0, x=k-1 over k), along with two new fixed points

> #f'(x)=kx-2k^{3}x $+ 6k^3x^2 - 2k^2x - 4k^3x^3$

> #We need f''x<1

> $x41 := \frac{\left(0.5 \cdot k + 0.5 + \frac{\text{sqrt}\left(k^2 - 2 \cdot k - 3\right)}{2}\right)}{k}$

$$x41 := \frac{0.5\, k + 0.5 + \frac{\sqrt{k^2-2k-3}}{2}}{k} \quad \textbf{(30)}$$

> $x42 := \frac{\left(0.5 \cdot k + 0.5 - \frac{\text{sqrt}\left(k^2 - 2 \cdot k - 3\right)}{2}\right)}{k}$

$$x42 := \frac{0.5\,k + 0.5 - \frac{\sqrt{k^2 - 2\,k - 3}}{2}}{k} \quad \textbf{(31)}$$

> $f\_prime := k \cdot x - 2 \cdot k^3 \cdot x + 6 \cdot k^3 \cdot x^2 - 2 \cdot k^2 \cdot x - 4 \cdot k^3 \cdot x^3;$

$$f\_prime := -4\,k^3\,x^3 + 6\,k^3\,x^2 - 2\,k^3\,x - 2\,k^2\,x + k\,x \quad \textbf{(32)}$$

> *#We need to plug in x41 and x42 for x and solve for f_prime<1 -> this will give us our condition for stable fixed points*

> $f\_prime\_1 := -4 \cdot k^3 \cdot x41^3 + 6 \cdot k^3 \cdot x41^2 - 2 \cdot k^3 \cdot x41 - 2 \cdot k^2 \cdot x41 + k \cdot x41$

$$f\_prime\_1 := -4\left(0.5\,k + 0.5 + \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right)^3 + 6\,k\left(0.5\,k + 0.5 + \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right)^2 - 2\,k^2\left(0.5\,k + 0.5 + \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right) - 2\,k\left(0.5\,k + 0.5 + \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right) + 0.5\,k + 0.5 + \frac{\sqrt{k^2 - 2\,k - 3}}{2} \quad \textbf{(33)}$$

> $f\_prime\_2 := -4 \cdot k^3 \cdot x42^3 + 6 \cdot k^3 \cdot x42^2 - 2 \cdot k^3 \cdot x42 - 2 \cdot k^2 \cdot x42 + k \cdot x42$

$$f\_prime\_2 := -4\left(0.5\,k + 0.5 - \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right)^3 + 6\,k\left(0.5\,k + 0.5 - \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right)^2 - 2\,k^2\left(0.5\,k + 0.5 - \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right) - 2\,k\left(0.5\,k + 0.5 - \frac{\sqrt{k^2 - 2\,k - 3}}{2}\right) + 0.5\,k + 0.5 - \frac{\sqrt{k^2 - 2\,k - 3}}{2} \quad \textbf{(34)}$$

> $solve(f\_prime\_1, k)$

$$-1., -1.023467373, 2.261733686 + 0.1190881984\,I, 2.261733686 - 0.1190881984\,I \quad \textbf{(35)}$$

> $solve(f\_prime\_2, k)$

$$-1. \quad \textbf{(36)}$$

> *#For k between 1-4, we need k to be equal to above at bifurcation*

> $f\_4 := (k^2 \cdot x - k^2 \cdot x^2) \cdot (1 - k \cdot x + k \cdot x^2)$

$$f\_4 := (-k^2\,x^2 + k^2\,x)\,(k\,x^2 - k\,x + 1) \quad \textbf{(37)}$$

> $Orb(f\_4, x, 0.5, -1, -1.023467373)$

$$[0.5] \quad \textbf{(38)}$$

>