```
> # OK to post hw
> #Anne Somalwar, hw9, 10.4.2021
>
> read "C:/Users/aks238/OneDrive - Rutgers University/Documents/M9.txt"
>
> #1
  evalf(Orb(2·x·(1 − x), x, 0.5, 1000, 1010));
```

[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000] **(1)**

```
> SFP(2·x·(1 − x), x)
```

[0.5000000000] **(2)**

```
> evalf(Orb(2.5·x·(1 − x), x, 0.5, 1000, 1010));
```

[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000] **(3)**

```
> SFP(2.5·x·(1 − x), x)
```

[0.6000000000] **(4)**

```
> evalf(Orb(3.1·x·(1 − x), x, 0.5, 1000, 1010));
```

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203] **(5)**

```
> SFP(3.1·x·(1 − x), x)
```

[ ] **(6)**

> $evalf\left(Orb\left(\frac{(4+x)}{3+x}, x, 0.5, 1000, 1010\right)\right);$

[1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978, 1.236067977, 1.236067978] **(7)**

> $SFP\left(\frac{(4+x)}{3+x}, x\right)$

[1.236067977] **(8)**

> $evalf\left(Orb\left(\frac{(3+x)}{4+x}, x, 0.5, 1000, 1010\right)\right);$

[0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475, 0.7912878475] **(9)**

> $SFP\left(\frac{(3+x)}{4+x}, x\right)$

[0.791287848] **(10)**

> $evalf\left(Orb\left(\frac{(3+x+x^2)}{4+x+2\cdot x^2}, x, 0.5, 1000, 1010\right)\right);$

[0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591, 0.7351392591] **(11)**

> $SFP\left(\frac{(3+x+x^2)}{4+x+2\cdot x^2}, x\right)$

[0.7351392587] **(12)**

>

>

> #2

>

> $solve\left(x=\frac{(x+a)}{x+b}, x\right)$

$$-\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4\,a-2\,b+1}}{2}, -\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4\,a-2\,b+1}}{2} \quad \textbf{(13)}$$

>

>

> $diff\left(\frac{(x+a)}{x+b}, x\right)$

$$\frac{1}{x+b}-\frac{x+a}{(x+b)^2} \quad \textbf{(14)}$$

> $C1 := eval\left(\%, x=-\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4\,a-2\,b+1}}{2}\right);$

$$C1 := \frac{1}{\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4\,a-2\,b+1}}{2}}-\frac{-\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4\,a-2\,b+1}}{2}+a}{\left(\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2+4\,a-2\,b+1}}{2}\right)^2} \quad \textbf{(15)}$$

> $diff\left(\frac{(x+a)}{x+b}, x\right)$

$$\frac{1}{x+b}-\frac{x+a}{(x+b)^2} \quad \textbf{(16)}$$

> $C2 := eval\left(\%, x=-\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4\,a-2\,b+1}}{2}\right);$

(17)

$$C2 := \frac{1}{\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4a-2b+1}}{2}} - \frac{-\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4a-2b+1}}{2}+a}{\left(\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2+4a-2b+1}}{2}\right)^2} \quad \textbf{(17)}$$

> #C(a,b) is represented by C1 and C2.

> #a=1, b=2

> eval(C1, a = 1)

$$\frac{1}{\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2-2b+5}}{2}} - \frac{-\frac{b}{2}+\frac{3}{2}+\frac{\sqrt{b^2-2b+5}}{2}}{\left(\frac{b}{2}+\frac{1}{2}+\frac{\sqrt{b^2-2b+5}}{2}\right)^2} \quad \textbf{(18)}$$

> eval(%, b = 2)

$$\frac{1}{\frac{3}{2}+\frac{\sqrt{5}}{2}} - \frac{\frac{1}{2}+\frac{\sqrt{5}}{2}}{\left(\frac{3}{2}+\frac{\sqrt{5}}{2}\right)^2} \quad \textbf{(19)}$$

> evalf(%)

$$0.1458980339 \quad \textbf{(20)}$$

> eval(C2, a = 1)

$$\frac{1}{\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2-2b+5}}{2}} - \frac{-\frac{b}{2}+\frac{3}{2}-\frac{\sqrt{b^2-2b+5}}{2}}{\left(\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2-2b+5}}{2}\right)^2} \quad \textbf{(21)}$$

> eval(%, b = 2)

$$\frac{1}{\frac{3}{2}-\frac{\sqrt{5}}{2}} - \frac{\frac{1}{2}-\frac{\sqrt{5}}{2}}{\left(\frac{3}{2}-\frac{\sqrt{5}}{2}\right)^2} \quad \textbf{(22)}$$

> evalf(%)

$$6.854101940 \quad \textbf{(23)}$$

> #This implies that there is one stable fixed point when a=1, b=2

>

> #Check against Orb:

>

$$Orb\left(\frac{(x+1)}{x+2}, x, 0.5, 1000, 1010\right);$$

[0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, **(24)** 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888, 0.6180339888]

> *#Check against FP*

> $FP\left(\frac{(x+1)}{x+2}, x\right)$

$$[-1.618033988, 0.6180339880] \quad (25)$$

> $SFP\left(\frac{(x+1)}{x+2}, x\right)$

$$[0.6180339880] \quad (26)$$

> *#a=2, b=3*

>

> $eval(C1, a=2)$:

> $eval(\%, b=3)$

$$\frac{1}{2+\frac{\sqrt{12}}{2}} - \frac{1+\frac{\sqrt{12}}{2}}{\left(2+\frac{\sqrt{12}}{2}\right)^2} \quad (27)$$

> $evalf(\%)$

$$0.0717967697 \quad (28)$$

> $eval(C2, a=2)$

$$\frac{1}{\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2-2b+9}}{2}} - \frac{-\frac{b}{2}+\frac{5}{2}-\frac{\sqrt{b^2-2b+9}}{2}}{\left(\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2-2b+9}}{2}\right)^2} \quad (29)$$

> $eval(\%, b=3)$

$$\frac{1}{2-\frac{\sqrt{12}}{2}} - \frac{1-\frac{\sqrt{12}}{2}}{\left(2-\frac{\sqrt{12}}{2}\right)^2} \quad (30)$$

> $evalf(\%)$

$$13.92820327 \quad (31)$$

> *#This implies that there is one stable fixed point when a=2, b=3*
#Check against Orb
$Orb\left(\frac{(x+2)}{x+3}, x, 0.5, 1000, 1010\right)$;

[0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076, 0.7320508076] **(32)**

> $FP\left(\frac{(x+2)}{x+3}, x\right)$

$$[-2.732050808, 0.732050808] \quad (33)$$

> $SFP\left(\frac{(x+2)}{x+3}, x\right)$

$$[0.732050808] \quad (34)$$

>

> #a=12, b=17

>

> $eval(C1, a=12)$:

> $eval(\%, b=17)$

$$\frac{1}{9+\frac{\sqrt{304}}{2}} - \frac{4+\frac{\sqrt{304}}{2}}{\left(9+\frac{\sqrt{304}}{2}\right)^2} \quad (35)$$

> $evalf(\%)$

$$0.01592760652 \quad (36)$$

> $eval(C2, a=12)$

$$\frac{1}{\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2-2b+49}}{2}} - \frac{-\frac{b}{2}+\frac{25}{2}-\frac{\sqrt{b^2-2b+49}}{2}}{\left(\frac{b}{2}+\frac{1}{2}-\frac{\sqrt{b^2-2b+49}}{2}\right)^2} \quad (37)$$

> $eval(\%, b=17)$

$$\frac{1}{9-\frac{\sqrt{304}}{2}} - \frac{4-\frac{\sqrt{304}}{2}}{\left(9-\frac{\sqrt{304}}{2}\right)^2} \quad (38)$$

> $evalf(\%)$

$$62.78407147 \quad (39)$$

>

> #This implies that there is a stable fixed point when a=12, b=17
 #Check against Orb
 $Orb\left(\frac{(x+12)}{x+17}, x, 0.5, 1000, 1010\right)$;

[0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871, 0.7177978871] (40)

> #Check against FP

> $FP\left(\frac{(x+12)}{x+17}, x\right)$

$$[-16.71779789, 0.717797888] \quad (41)$$

> $SFP\left(\frac{(x+12)}{x+17}, x\right)$

$$[0.717797888] \quad (42)$$

>

>

> #3

>

> $solve(x = k \cdot x \cdot (1 - x), x)$

$$0, \frac{k-1}{k} \quad (43)$$

> $diff(k \cdot x \cdot (1 - x), x)$

$$k(1-x) - kx \quad (44)$$

> $eval(\%, x = 0)$

$$k \quad (45)$$

> # f ' (0) = k. Since k>=1, f ' (0) is never less than 1, so 0 cannot be stable.
> $diff(k \cdot x \cdot (1 - x), x)$:
> $eval\left(\%, x = \frac{k-1}{k}\right)$

$$k\left(1 - \frac{k-1}{k}\right) - k + 1 \quad (46)$$

> $solve\left(-1 < k\left(1 - \frac{k-1}{k}\right) - k + 1 < 1, k\right);$

$$(1, 3) \quad (47)$$

> #These are the values where the other fixed point is stable. This suggests 3 is the first bifurcation point.

(48)

> $Orb(2.9 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.6551724126, 0.6551724150, 0.6551724126, 0.6551724150, 0.6551724126, 0.6551724150, 0.6551724126, 0.6551724150, 0.6551724126, 0.6551724150, 0.6551724126, 0.6551724150] (49)

(50)

> $Orb(3.1 \cdot x \cdot (1 - x), x, 0.5, 1000, 1010)$

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, (51) 0.7645665203]

(52)

> #So 3 is a bifucation point.

(53)

>

(54)

> #4

solve(x = k·(k·x·(1 − x))·(1 − (k·x·(1 − x))), x)

$$0, \frac{k-1}{k}, \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}, \frac{\frac{k}{2}+\frac{1}{2}-\frac{\sqrt{k^2-2k-3}}{2}}{k} \quad \textbf{(55)}$$

> diff(k·(k·x·(1 − x))·(1 − (k·x·(1 − x))), x)

$$k^2 (1-x) (1-k x (1-x)) - k^2 x (1-k x (1-x)) + k^2 x (1-x) (-k (1-x) + k x) \quad \textbf{(56)}$$

$$> B1 := eval\left(\%, x = \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}\right)$$

$$B1 := k^2 \left(1 - \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}\right) \left(1 - \left(\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}\right) \left(1 - \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}\right)\right) - k \left(\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}\right) \left(1 - \left(\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}\right) \left(1 - \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}\right)\right) + k \left(\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}\right) \left(1 - \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}\right) \left(-k \left(1 - \frac{\frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}}{k}\right) + \frac{k}{2}+\frac{1}{2}+\frac{\sqrt{k^2-2k-3}}{2}\right) \quad \textbf{(57)}$$

> diff(k·(k·x·(1 − x))·(1 − (k·x·(1 − x))), x) :

$$> B2 := eval\left(\%, x = \frac{\frac{k}{2}+\frac{1}{2}-\frac{\sqrt{k^2-2k-3}}{2}}{k}\right)$$

$$B2 := k^2 \left(1 - \frac{\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right) \left(1 - \left(\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \frac{\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right)\right) - k \left(\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \left(\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \frac{\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right)\right) + k \left(\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \frac{\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right) \left(-k \left(1 - \frac{\frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right) + \frac{k}{2} + \frac{1}{2} - \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \quad \textbf{(58)}$$

> *#These new fixed points are stable when the absolute value of B1 and B2 are less than 1.*

>

>

>

> $solve\left(-1 < k^2 \left(1 - \frac{\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right) \left(1 - \left(\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \frac{\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right)\right) - k \left(\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \left(\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \frac{\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right)\right) + k \left(\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}\right) \left(1 - \frac{\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right) \left(-k \left(1 - \frac{\frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}}{k}\right) + \frac{k}{2} + \frac{1}{2} + \frac{\sqrt{k^2 - 2k - 3}}{2}\right) < 1, k\right)$

$$\left(-\frac{(1-\sqrt{6}+\sqrt{2})^3}{8}+\frac{3(1-\sqrt{6}+\sqrt{2})^2}{8}+\frac{3}{4}-\frac{17\sqrt{6}}{8}+\frac{17\sqrt{2}}{8},-1\right),\left(3,-\frac{(1+\sqrt{6}+\sqrt{2})^3}{8}+\frac{3(1+\sqrt{6}+\sqrt{2})^2}{8}+\frac{3}{4}+\frac{17\sqrt{6}}{8}+\frac{17\sqrt{2}}{8}\right) \quad (59)$$

> $evalf\left(-\frac{(1+\sqrt{6}+\sqrt{2})^3}{8}+\frac{3(1+\sqrt{6}+\sqrt{2})^2}{8}+\frac{3}{4}+\frac{17\sqrt{6}}{8}+\frac{17\sqrt{2}}{8}\right)$

3.449489733 **(60)**

> *#This is the second bifurcation point.*

> $Orb(3.4\cdot(3.4\cdot x\cdot(1-x))\cdot(1-(3.4\cdot x\cdot(1-x))), x, 0.5, 1000, 1010)$

[0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476, 0.4519632476] **(61)**

> $Orb(3.5\cdot(3.5\cdot x\cdot(1-x))\cdot(1-(3.5\cdot x\cdot(1-x))), x, 0.5, 1000, 1010)$

[0.5008842103, 0.3828196828, 0.5008842103, 0.3828196828, 0.5008842103, 0.3828196828, 0.5008842103, 0.3828196828, 0.5008842103, 0.3828196828, 0.5008842103, 0.3828196828] **(62)**

>