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> # Max Mekhanikov - RUID 184004391 - HW 8 - Okay to post
  # Question 1
>
  # Orb(f,x,x0,K1,K2): Inputs an expression f in x (describing) a function of x, an initial point, x0, and a positive integer K, outputs
  # the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
  #Orb(2*x*(1-x),x,0.4,1000,2000);

  Orb := proc(f, x, x0, K1, K2) local x1, i, L :
  x1 := x0 :
  for i from 1 to K1 do
   x1 := subs(x=x1, f) :  #we don't record the first values of K1, since we are interested in the long-time behavior of the orbit
  od:

  L := [x1] :

  for i from K1 to K2 do
   x1 := subs(x=x1, f) :   #we compute the next member of the orbit
   L := [op(L), x1] : #we append it to the list
  od:

  L : #that's the output

  end:
```

> $Orb\left(\frac{1+8\cdot x}{4+x}, x, 1, 1, 25\right);$

$$\left[\frac{9}{5}, \frac{77}{29}, \frac{645}{193}, \frac{5353}{1417}, \frac{44241}{11021}, \frac{364949}{88325}, \frac{3007917}{718249}, \frac{24781585}{5880913}, \frac{204133593}{48305237}, \frac{1681373981}{397354541}, \frac{13848346389}{3270792145}, \frac{114057563257}{26931514969}, \frac{939392021025}{221783623133}, \frac{7736919791333}{1826526513557}, \frac{63721884844221}{15043025845561}, \frac{524818104599329}{123893988226465}, \frac{4322438825021097}{1020394057505189}, \frac{35599904657673965}{8404015055041853}, \frac{293203252316433573}{69215964877841377}, \frac{2414841983409309961}{570067111827799081}, \frac{19888802979102278769}{4695110430720506285}, \frac{163805534263538736437}{38669244701984303909}, \frac{1349113518810294195405}{318482513071475952073}, \frac{11111390663553829515313}{2623043571096198003697}, \frac{91514168879526834126201}{21603564947938621530101}, \frac{753716915984153294539709}{177928428671281320246605}\right] \quad (1)$$

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> # Maple cannot compute all 1000 terms using this method and results in the error "[Length of output exceeds limit of 1000000]". However,
  # when computing the first 20 terms, it becomes evident that this sequence reaches the steady state of roughly 4.236067. We can prove this
  # is true by finding the derivative of f(x) and plugging in our equilibrium point value, x bar, in order to see if the absolute value of # # the resulting
  expression is less than 1.
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> $fprime := diff\left(\frac{1+8\cdot x}{4+x}, x\right)$

$$fprime := \frac{8}{4+x} - \frac{1+8x}{(4+x)^2} \quad (2)$$

> $\frac{8}{4+4.23607} - \frac{1+8\cdot 4.23607}{(4+4.23607)^2}$

$$0.4570057198 \quad (3)$$

> # Here we see f'(x bar) is less than 1 which is the condition required to show x bar = 4.23067 is a stable equilibrium point.

Question 2

> # K=1

> $Orb(x \cdot (1 - x), x, 0.5, 900, 1000)$

[0.001100619364, 0.001099408001, 0.001098199303, 0.001096993261, 0.001095789867, 0.001094589112, 0.001093390987, 0.001092195483, 0.001091002592, 0.001089812305, 0.001088624614, 0.001087439510, 0.001086256985, 0.001085077031, 0.001083899639, 0.001082724801, 0.001081552508, 0.001080382752, 0.001079215525, 0.001078050819, 0.001076888625, 0.001075728936, 0.001074571743, 0.001073417039, 0.001072264815, 0.001071115063, 0.001069967775, 0.001068822944, 0.001067680562, 0.001066540620, 0.001065403111, 0.001064268027, 0.001063135361, 0.001062005104, 0.001060877249, 0.001059751789, 0.001058628715, 0.001057508020, 0.001056389697, 0.001055273738, 0.001054160135, 0.001053048881, 0.001051939969, 0.001050833391, 0.001049729140, 0.001048627209, 0.001047527590, 0.001046430276, 0.001045335260, 0.001044242534, 0.001043152092, 0.001042063926, 0.001040978029, 0.001039894394, 0.001038813014, 0.001037733882, 0.001036656990, 0.001035582332, 0.001034509901, 0.001033439690, 0.001032371692, 0.001031305901, 0.001030242309, 0.001029180910, 0.001028121697, 0.001027064663, 0.001026009801, 0.001024957105, 0.001023906568, 0.001022858183, 0.001021811944, 0.001020767844, 0.001019725877, 0.001018686036, 0.001017648315, 0.001016612707, 0.001015579206, 0.001014547805, 0.001013518498, 0.001012491278, 0.001011466139, 0.001010443075, 0.001009422080, 0.001008403147, 0.001007386270, 0.001006371443, 0.001005358660, 0.001004347914, 0.001003339199, 0.001002332509, 0.001001327839, 0.001000325182, 0.0009993245315, 0.0009983258820, 0.0009973292274, 0.0009963345618, 0.0009953418792, 0.0009943511737, 0.0009933624394, 0.0009923756705, 0.0009913908610, 0.0009904080051] **(4)**

> # Here all 1000 values were calculated but only the last 100 are displayed in order to save space. As we can see, the values continue to
> # decline and never settle on an equilibrium point.

K=2

> $Orb(2 \cdot x \cdot (1 - x), x, 0.5, 1, 100)$

[0.50, 0.5000, 0.50000000, 0.5000000000] **(5)**

> # Just as in the previous case, all 1000 values were calculated but this time only the first 100 are shown. It is clear that 0.5 is an
> # equilibrium point for this recurrence and can be checked for stability using the same procedure as in question number one.

$diff(2 \cdot x \cdot (1 - x), x); 2 - 4(0.5);$

$$2 - 4x$$

$$0.$$ **(6)**

>

> # x bar = 0.5 is therefore a stable equilibrium point

```
# K=2.5
> Orb(2.5·x·(1 − x), x, 0.5, 1, 50)
```

[0.625, 0.5859375, 0.6065368652, 0.5966247410, 0.6016591485, 0.5991635438, 0.6004164790, 0.5997913268, 0.6001042278, 0.5999478590, 0.6000260638, 0.5999869665, 0.6000065162, 0.5999967418, 0.6000016290, 0.5999991855, 0.6000004072, 0.5999997965, 0.6000001018, 0.5999999490, 0.6000000255, 0.5999999872, 0.6000000065, 0.5999999968, 0.6000000015, 0.5999999992, 0.6000000005, 0.5999999998, 0.6000000000] (7)

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> # All 1000 values were calculated and the recurrence remains at 0.60 until the end, making this the equilibrium point which can also be
  # checked for stability. Unlike k=2 however, this recurrence took several terms to reach its equil value rather than remaining at the same
  # value from x=0.

  diff(2.5·x·(1 − x), x); 2.5 − 5(0.6);
```

$$2.5 - 5.0\,x$$
$$-0.5 \qquad (8)$$

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> # |f'(x bar)| < 1 therefore 0.60 is a stable equilibrium point.

  # K = 3.1
> Orb(3.1·x·(1 − x), x, 0.5, 1, 100)
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[0.775, 0.5405625, 0.7698995192, 0.5491781734, 0.7675026726, 0.5531711926, 0.7662357553, 0.5552674201, 0.7655310881, 0.5564290480, 0.7651288638, 0.5570907254, 0.7648960122, 0.5574733184, 0.7647601349, 0.5576964201, 0.7646804816, 0.5578271524, 0.7646336632, 0.5579039753, 0.7646061018, 0.5579491938, 0.7645898618, 0.5579758355, 0.7645802878, 0.5579915410, 0.7645746417, 0.5580008029, 0.7645713114, 0.5580062657, 0.7645693466, 0.5580094885, 0.7645681875, 0.5580113900, 0.7645675037, 0.5580125116, 0.7645671004, 0.5580131731, 0.7645668623, 0.5580135637, 0.7645667218, 0.5580137941, 0.7645666391, 0.5580139298, 0.7645665901, 0.5580140101, 0.7645665613, 0.5580140573, 0.7645665445, 0.5580140848, 0.7645665346, 0.5580141013, 0.7645665287, 0.5580141109, 0.7645665250, 0.5580141168, 0.7645665231, 0.5580141202, 0.7645665219, 0.5580141220, 0.7645665210, 0.5580141236, 0.7645665206, 0.5580141242, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203] (9)

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> # This recurrence does not have a fixed equilibrium point, but rather oscillates between 0.7645 and 0.5580 up until the 1000th term and beyond.

  # K = 3.5
> Orb(3.5·x·(1 − x), x, 0.5, 1, 100)
```

[0.875, 0.3828125, 0.8269348143, 0.5008976952, 0.8749971794, 0.3828199039, 0.8269408878, 0.5008837956, 0.8749972662, 0.3828196760, 0.8269407010, 0.5008842230, 0.8749972634, 0.3828196834, 0.8269407069, 0.5008842097, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637] (10)

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> # Just is the previous case, this recurrence does not have a fixed equilibrium point either. However, it does oscillate between four
  # values rather than only two this time. Those values are 0.8749, 0.3828, 0.8269, and 0.5008 in a repeating sequence.
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Question 3

>

$$Orb\left(1 + \frac{8}{9}\cdot x, x, 0.5, 1, 200\right);$$

[1.444444444, 2.283950617, 3.030178326, 3.693491845, 4.283103862, 4.807203433, 5.273069718, 5.687173083, 6.055264963, 6.382457745, 6.673295773, 6.931818465, 7.161616413, 7.365881256, 7.547450005, 7.708844449, 7.852306177, 7.979827713, 8.093180189, 8.193937946, 8.283500396, 8.363111463, 8.433876856, 8.496779428, 8.552692825, 8.602393622, 8.646572108, 8.685841874, 8.720748332, 8.751776295, 8.779356707, 8.803872628, 8.825664558, 8.845035163, 8.862253478, 8.877558647, 8.891163242, 8.903256215, 8.914005524, 8.923560466, 8.932053748, 8.939603332, 8.946314073, 8.952279176, 8.957581490, 8.962294658, 8.966484140, 8.970208124, 8.973518332, 8.976460740, 8.979076213, 8.981401078, 8.983467625, 8.985304556, 8.986937383, 8.988388785, 8.989678920, 8.990825707, 8.991845073, 8.992751176, 8.993556601, 8.994272534, 8.994908919, 8.995474595, 8.995977418, 8.996424372, 8.996821664, 8.997174812, 8.997488722, 8.997767753, 8.998015780, 8.998236249, 8.998432221, 8.998606419, 8.998761261, 8.998898899, 8.999021244, 8.999129995, 8.999226662, 8.999312588, 8.999388967, 8.999456860, 8.999517209, 8.999570852, 8.999618535, 8.999660920, 8.999698596, 8.999732085, 8.999761853, 8.999788314, 8.999811835, 8.999832742, 8.999851326, 8.999867845, 8.999882529, 8.999895581, 8.999907183, 8.999917496, 8.999926663, 8.999934812, 8.999942055, 8.999948493, 8.999954216, 8.999959303, 8.999963825, 8.999967844, 8.999971417, 8.999974593, 8.999977416, 8.999979925, 8.999982156, 8.999984139, 8.999985901, 8.999987468, 8.999988860, 8.999990098, 8.999991198, 8.999992176, 8.999993045, 8.999993818, 8.999994505, 8.999995116, 8.999995659, 8.999996141, 8.999996570, 8.999996951, 8.999997290, 8.999997591, 8.999997859, 8.999998097, 8.999998308, 8.999998496, 8.999998663, 8.999998812, 8.999998944, 8.999999061, 8.999999165, 8.999999258, 8.999999340, 8.999999413, 8.999999478, 8.999999536, 8.999999588, 8.999999634, 8.999999675, 8.999999711, 8.999999743, 8.999999772, 8.999999797, 8.999999820, 8.999999840, 8.999999858, 8.999999874, 8.999999888, 8.999999900, 8.999999911, 8.999999921, 8.999999930, 8.999999938, 8.999999945, 8.999999951, 8.999999956, 8.999999961, 8.999999965, 8.999999969, 8.999999972, 8.999999975, 8.999999978, 8.999999980, 8.999999982, 8.999999984, 8.999999986, 8.999999988, 8.999999989, 8.999999990, 8.999999991, 8.999999992, 8.999999993, 8.999999994, 8.999999995, 8.999999996] **(11)**

> # The recurrence remains at 8.99999 until x=1000, making that value the steady state equilibrium point.

$$diff\left(1 + \frac{8}{9}\cdot x, x\right);$$

$$\frac{8}{9} \qquad (12)$$

> # f'(x bar)| < 1 therefore 8.999 is a stable equilibrium point.

$$X_n = \frac{a_1 x_{n-1} + a_2 x_{n-2}}{a_5 x_{n-1} + a_7 x_{n-2}}$$

$a_1 = 1$, $a_2 = 8$

$a_5 = 1$, $a_7 = 9$

$$X_n = \frac{x_{n-1} + 8(x_{n-2})}{x_{n-1} + 9(x_{n-2})}$$

$$x_n = 1 + \frac{8}{9}(x_{n-2})$$

$X_0 = 0.5$, $x_1 = 0.7$

$$x_n = f(x_{n-2})$$

$f(x) = 1 + \frac{8}{9}(x)$, $x_0 = 0.5$

orb(f, x, x_0, k1, k2)

orb($1 + \frac{8}{9}x$, x, 0.5, 1, 1000);