```
> #Do not post
  #Nikita John, Assignment 8
  # October 4th, 2021
> #M9.txt: Maple Code for "Dynamical models in Biology" (Math 336) taught by Dr. Z., Lecture 9

  Help9 :=proc( ) :
       print( ` Orb(f,x,x0,K1,K2), Orb2D(f,x,x0,K) , FP(f,x) , SFP(f,x) , Comp(f,x)  `) :end:


       #Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point,
       x0, and a positive integer K, outputs
  #the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
  #Orb(2*x*(1-x),x,0.4,1000,2000);
  Orb :=proc( f, x, x0, K1, K2 ) local x1, i, L :
  x1 := x0 :
  for i from 1 to K1 do
   x1 := subs(x=x1, f ) :
       #we don't record the first values of K1, since we are interested in the long-time behavior of
       the orbit
  od:

  L := [x1] :

  for i from K1 to K2 do
   x1 := subs(x=x1, f ) :   #we compute the next member of the orbit
   L := [op(L), x1] : #we append it to the list
  od:

  L : #that's the output

  end:

  #Orb2D(f,x,x0,K): 2D version of Orb(f,x,x0,0,K), just for illustration
  Orb2D :=proc( f, x, x0, K) local L, L1, i :
  L := Orb( f, x, x0, 0, K) :
  L1 := [ [L[1], 0], [L[1], L[2]], [L[2], L[2]] ] :
  for i from 3 to nops(L) do
   L1 := [op(L1), [L[i-1], L[i]], [L[i], L[i]]] :
  od:
  L1 :
  end:


  #FP(f,x): The list of fixed points of the map x->f where f is an expression in x. Try:
  #FP(2*x*(1-x),x);
  FP :=proc( f, x)
  evalf ( [solve( f=x) ]) :
  end:
```

```
#SFP(f,x): The list of stable fixed points of the map x->f where f is an expression in x. Try:
#SFP(2*x*(1-x),x);
SFP :=proc(f, x) local L, i, f1, pt, Ls :
L := FP(f, x) : #The list of fixed points (including complex ones)

Ls := [ ] :       #Ls is the list of stable fixed points, that starts out as the empty list

f1 := diff(f, x) :  #The derivative of the function f w.r.t. x

for i from 1 to nops(L) do
pt := L[i] :

if abs(subs(x=pt, f1)) < 1 then

 Ls := [op(Ls), pt] :  # if pt,  is stable we add it to the list of stable points

fi:

od:

Ls :   #The last line is the output

end:

#Comp(f,x): f(f(x))
Comp :=proc(f, x) : normal(subs(x=f, f)) :end:
```

```
> #a1 = 1, a2 = 8, a3 = 4, a4 = 1
```

$$f1 := \frac{(1 + 8 \cdot x)}{(4 + x)};$$

$$f1 := \frac{1 + 8\,x}{4 + x} \quad (1)$$

```
> #1
evalf(Orb(f1, x, 1, 1, 1000));
```

#We can see by inspection that one fixed point is 4.236067977 because it repeats over and over for most of the 1000 entries

[1.800000000, 2.655172414, 3.341968912, 3.777699365, 4.014245531, 4.131887914, (2)
4.187847112, 4.213900971, 4.225910184, 4.231420073, 4.233942658, 4.235096443,
4.235623928, 4.235865033, 4.235975229, 4.236025590, 4.236048606, 4.236059125,
4.236063932, 4.236066129, 4.236067133, 4.236067591, 4.236067801, 4.236067897,
4.236067941, 4.236067961, 4.236067970, 4.236067974, 4.236067976, 4.236067977,
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> $FP(f1, x);$
#Finding the fixed points gives us two total, one of which is the one seen repeated over and over above

$$[-0.236067977, 4.236067977] \quad \textbf{(3)}$$

> $SFP(f1, x);$
#As expected, the only stable fixed point is 4.236067977 since we see it repeat constantly

$$[4.236067977] \quad \textbf{(4)}$$

> *#2: For the sake of a concise code, only the last ten numbers in the 1000 point sequence will be printed for each situation. It should be enough to gauge the behavior.*

$F1 := x \cdot (1 - x) :$
$F2 := 2 \cdot x \cdot (1 - x) :$
$F3 := 2.5 \cdot x \cdot (1 - x) :$
$F4 := 3.1 \cdot x \cdot (1 - x) :$
$F5 := 3.5 \cdot x \cdot (1 - x) :$

> $Orb(F1, x, 0.5, 990, 1000);$
#The values are slowly decreasing with each successive entry, surprisingly

[0.001001327839, 0.001000325182, 0.0009993245315, 0.0009983258820, 0.0009973292274, **(5)**
0.0009963345618, 0.0009953418792, 0.0009943511737, 0.0009933624394,
0.0009923756705, 0.0009913908610, 0.0009904080051]

> $Orb(F2, x, 0.5, 990, 1000);$
#All the entries are 0.5, indicating that 0.5 must be a stable fixed point

[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, **(6)**
0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,
0.5000000000]

> $Orb(F3, x, 0.5, 900, 1000);$

#This time, all these entries are 0.6, so 0.6 may also be a stable fixed point (I did 100 just to see the behavior and all of them are 0.6, so it must be a stable fixed point)

[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, **(7)**
0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,
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> Orb(F4, x, 0.5, 990, 1000);
```

#The entries flip between 0.558 and 0.765, indicating these must be two fixed points, but its hard to tell if they're stable fixed points or not from this function alone.

[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, **(8)**
0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,
0.7645665203]

```
> Orb(F5, x, 0.5, 900, 1000);
```

#This time the entries vary between 0.38, 0.83, and 0.5, going in a 0.5, 0.83, 0.38, 0.83 pattern. This seems like the bifurcation with period 4, but I'm not sure since 0.83 repeats with every other element. (I did 100 entries to see more of the pattern)

[0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, **(9)**
0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827,
0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
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0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
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0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827,
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0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111,
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0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111,
0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637,
0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827,
0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062,
0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111,
0.8749972637]

```
> #3: I wrote my own program to solve non-linear second order recurrences. a1 = 1, a2 = 8, a5 = 1, a7 = 5
Orb2 := proc(n)
if n = 0 then
  0.5 :
elif n = 1 then
0.7 :
else
expand( ((n − 1) + 8·(n − 2)) / ((n − 1) + 5·(n-2)) ) :
fi:
end:
```

```
> evalf(seq(Orb2(i), i = 0 ..1000));

#the entries seem to approach a steady state, but not in the way that the previous first order non-linear recurrences do with the same value repeating itself
```

0.5, 0.7, 1., 1.428571429, 1.461538462, 1.473684211, 1.480000000, 1.483870968, **(10)**
1.486486486, 1.488372093, 1.489795918, 1.490909091, 1.491803279, 1.492537313,
1.493150685, 1.493670886, 1.494117647, 1.494505494, 1.494845361, 1.495145631,
1.495412844, 1.495652174, 1.495867769, 1.496062992, 1.496240602, 1.496402878,
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