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> #Deven Singh, Assignment 8
  # OK TO POST
  #Question 1

  #Orb(f,x,x0,K1,K2): Inputs an expression f in x (desccribing) a function of x, an initial point, x0,
       and a positive integer K, outputs
  #the values of x[n] from n=K1 to n=K2. Try: where x[n]=f(x[n-1]), . Try:
  #Orb(2*x*(1-x),x,0.4,1000,2000);
  Orb := proc(f, x, x0, K1, K2) local x1, i, L :
  x1 := x0 :
  for i from 1 to K1 do
   x1 := subs(x = x1, f) :
       #we don't record the first values of K1, since we are interested in the long-time behavior of
       the orbit
  od:

  L := [x1] :

  for i from K1 to K2 do
   x1 := subs(x = x1, f) :   #we compute the next member of the orbit
   L := [op(L), x1] : #we append it to the list
  od:

  L : #that's the output

  end:
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$$> \; evalf\left(Orb\left(\frac{(1+8\,x)}{(5+x)}, x, 1, 990, 1000\right)\right);$$

$$[3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, \qquad \textbf{(1)}$$
$$3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638]$$

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> # y = 3.02775638 is a steady state solution
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$$> \; evalf\left(Orb\left(\frac{(1+8\,x)}{(5+x)}, x, 2, 990, 1000\right)\right);$$

$$[3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, \qquad \textbf{(2)}$$
$$3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638]$$

$$> \; evalf\left(Orb\left(\frac{(1+8\,x)}{(5+x)}, x, 0, 990, 1000\right)\right);$$

$$[3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, \qquad \textbf{(3)}$$
$$3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638, 3.302775638]$$

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> # y = 3.02775638 is stable
> #Question 2
> evalf(Orb(x·(1 − x), x, 0.5, 950, 1000));
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$$[0.001043152092, 0.001042063926, 0.001040978029, 0.001039894394, 0.001038813014, \qquad \textbf{(4)}$$
$$0.001037733882, 0.001036656990, 0.001035582332, 0.001034509901, 0.001033439690,$$
$$0.001032371692, 0.001031305901, 0.001030242309, 0.001029180910, 0.001028121697,$$

$$0.001027064663, 0.001026009801, 0.001024957105, 0.001023906568, 0.001022858183,$$
$$0.001021811944, 0.001020767844, 0.001019725877, 0.001018686036, 0.001017648315,$$
$$0.001016612707, 0.001015579206, 0.001014547805, 0.001013518498, 0.001012491278,$$
$$0.001011466139, 0.001010443075, 0.001009422080, 0.001008403147, 0.001007386270,$$
$$0.001006371443, 0.001005358660, 0.001004347914, 0.001003339199, 0.001002332509,$$
$$0.001001327839, 0.001000325182, 0.0009993245315, 0.0009983258820,$$
$$0.0009973292274, 0.0009963345618, 0.0009953418792, 0.0009943511737,$$
$$0.0009933624394, 0.0009923756705, 0.0009913908610, 0.0009904080051]$$

> $evalf(Orb(2 \cdot x \cdot (1 - x), x, 0.5, 950, 1000));$

$[0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000, 0.5000000000,$ **(5)**
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$0.5000000000]$

> $evalf(Orb(2.5 \cdot x * (1 - x), x, 0.5, 950, 1000));$

$[0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000, 0.6000000000,$ **(6)**
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$0.6000000000]$

> $evalf(Orb(3.1 \cdot x * (1 - x), x, 0.5, 950, 1000));$

$[0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203,$ **(7)**
$0.5580141245, 0.7645665203, 0.5580141245, 0.7645665203, 0.5580141245,$
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0.7645665203]

> $evalf(Orb(3.5 \cdot x * (1 - x), x, 0.5, 950, 1000));$

[0.3828196827, 0.8269407062, 0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, **(8)**

0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111,

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0.5008842111, 0.8749972637, 0.3828196827, 0.8269407062, 0.5008842111,

0.8749972637]

> *#There is a sudden qualitative change in behavior at k=2 when the function stops decreasing over time and remains steady at 0.5. k=2.5, the steady state changes from 0.5 to 0.6. At k= 3.1, the function oscillates between .56 and .77. At k=3.5, the function oscillates between four values.*

> *#Question 3*

> $F := \textbf{proc}(a1, a2, a5, a7, c0, c1, n)\ \textbf{option}\ remember:$
   if $n = 0$ **then**
    $c0:$
   elif $n = 1$ **then**
    $c1:$
   else
    $expand\left(\dfrac{(a1 \cdot F(a1, a2, a5, a7, c0, c1, n - 1) + a2 \cdot F(a1, a2, a5, a7, c0, c1, n - 2))}{(a5 \cdot F(a1, a2, a5, a7, c0, c1, n - 1) + a7 \cdot F(a1, a2, a5, a7, c0, c1, n - 2))} \right)$
   fi:
   end:

> $evalf(seq(F(1, 8, 4, 9, 0.5, 0.7, i), i = 950..1000));$

0.6923076923, 0.6923076922, 0.6923076922, 0.6923076923, 0.6923076923, 0.6923076922, **(9)**

0.6923076922, 0.6923076923, 0.6923076923, 0.6923076922, 0.6923076922,

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> *# The steady state is 0.6923076923*

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