```
> #OK to post homework
  #Shreya Ghosh, 09-27-2021, Assignment 7
> read "/Users/shreyaghosh/Documents/M7.txt"
> Help7( )
GR(p,i,N), GRt(p,i,N),  GRm(N,p), OneStepMarkov(P,i), MarkovTrip(P,K), StSa(P,K) ,  StS(P), StSp (1)
    (P,K), RandSM(N)
> #1.
  GRt(0.5, 1, 20)
                                        [1, 175]                                        (2)
> EstGR := proc(p, i, N, K) local n, wincounter, W, duration :
  n := 0 :
  wincounter := 0 :
  duration := 0 :
  do
  W := GRt(p, i, N) :
  n := n + 1 :
  duration := duration + W[2] :
  if W[1] = 1 then
  wincounter := wincounter + 1 :
  else
  wincounter := wincounter :
  fi:
  until n = K :
  return [wincounter/K, duration/K] :
  end:
> evalf(EstGR(0.5, 3, 10, 3000))
                              [0.2936666667, 21.28266667]                              (3)
>
> #3.
  ExactFairGR := proc(i, N) :
  return [i/N, i(N - i)] :
  end:
>
> f := 1 :
  do
  evalf(EstGR(0.5, f, 20, 3000));
  f := f + 1 :
  until f = 20;
                              [0.05366666667, 20.01533333]
                                         f := 2
                              [0.09400000000, 36.04333333]
                                         f := 3
```

$$[0.1486666667, 50.47533333]$$
$$f := 4$$
$$[0.1930000000, 64.23600000]$$
$$f := 5$$
$$[0.2596666667, 76.97666667]$$
$$f := 6$$
$$[0.2883333333, 83.60066667]$$
$$f := 7$$
$$[0.3666666667, 92.61200000]$$
$$f := 8$$
$$[0.4140000000, 94.90333333]$$
$$f := 9$$
$$[0.4460000000, 99.79933333]$$
$$f := 10$$
$$[0.4886666667, 97.32600000]$$
$$f := 11$$
$$[0.5496666667, 99.83000000]$$
$$f := 12$$
$$[0.5996666667, 97.12000000]$$
$$f := 13$$
$$[0.6516666667, 91.54800000]$$
$$f := 14$$
$$[0.6840000000, 86.06333333]$$
$$f := 15$$
$$[0.7510000000, 73.61800000]$$
$$f := 16$$
$$[0.7920000000, 65.28533333]$$
$$f := 17$$
$$[0.8516666667, 50.98266667]$$
$$f := 18$$
$$[0.9060000000, 36.18000000]$$
$$f := 19$$
$$[0.9566666667, 16.37533333]$$
$$f := 20 \quad \textbf{(4)}$$

```
> g := 1 :
  do
  ExactFairGR(g, 20);
  g := g + 1 :
  until g = 20;
```

$$\left[\frac{1}{20}, 1\right]$$

$$g := 2$$

$$\left[\frac{1}{10}, 2\right]$$

$$g := 3$$

$$\left[\frac{3}{20}, 3\right]$$

$$g := 4$$

$$\left[\frac{1}{5}, 4\right]$$

$$g := 5$$

$$\left[\frac{1}{4}, 5\right]$$

$$g := 6$$

$$\left[\frac{3}{10}, 6\right]$$

$$g := 7$$

$$\left[\frac{7}{20}, 7\right]$$

$$g := 8$$

$$\left[\frac{2}{5}, 8\right]$$

$$g := 9$$

$$\left[\frac{9}{20}, 9\right]$$

$$g := 10$$

$$\left[\frac{1}{2}, 10\right]$$

$$g := 11$$

$$\left[\frac{11}{20}, 11\right]$$

$$g := 12$$

$$\left[\frac{3}{5}, 12\right]$$

$$g := 13$$

$$\left[\frac{13}{20}, 13\right]$$

$$g := 14$$

$$\left[\frac{7}{10}, 14\right]$$
$$g := 15$$
$$\left[\frac{3}{4}, 15\right]$$
$$g := 16$$
$$\left[\frac{4}{5}, 16\right]$$
$$g := 17$$
$$\left[\frac{17}{20}, 17\right]$$
$$g := 18$$
$$\left[\frac{9}{10}, 18\right]$$
$$g := 19$$
$$\left[\frac{19}{20}, 19\right]$$
$$g := 20 \quad \textbf{(5)}$$

\>

\> *#4.*
MarkovP := *RandSM*(10)

$$MarkovP := \left[\left[\frac{811}{5472}, \frac{407}{2736}, \frac{203}{5472}, \frac{25}{1824}, \frac{5}{228}, \frac{143}{1824}, \frac{941}{5472}, \frac{157}{2736}, \frac{263}{1824}, \frac{61}{342}\right], \quad \textbf{(6)}$$
$$\left[\frac{961}{4850}, \frac{147}{4850}, \frac{497}{4850}, \frac{10}{97}, \frac{3}{194}, \frac{494}{2425}, \frac{37}{970}, \frac{377}{4850}, \frac{487}{2425}, \frac{73}{2425}\right], \left[\frac{135}{4658}, \frac{32}{2329},\right.$$
$$\left.\frac{320}{2329}, \frac{3}{34}, \frac{133}{4658}, \frac{583}{4658}, \frac{151}{2329}, \frac{841}{4658}, \frac{707}{4658}, \frac{421}{2329}\right], \left[\frac{291}{2795}, \frac{463}{2795}, \frac{84}{559}, \frac{493}{2795},\right.$$
$$\left.\frac{11}{430}, \frac{127}{5590}, \frac{749}{5590}, \frac{1}{130}, \frac{201}{5590}, \frac{993}{5590}\right], \left[\frac{235}{5912}, \frac{751}{5912}, \frac{79}{1478}, \frac{339}{2956}, \frac{527}{5912}, \frac{911}{5912},\right.$$
$$\left.\frac{485}{5912}, \frac{391}{5912}, \frac{427}{2956}, \frac{191}{1478}\right], \left[\frac{196}{5515}, \frac{122}{1103}, \frac{678}{5515}, \frac{943}{5515}, \frac{28}{5515}, \frac{891}{5515}, \frac{35}{1103},\right.$$
$$\left.\frac{241}{5515}, \frac{829}{5515}, \frac{924}{5515}\right], \left[\frac{509}{3889}, \frac{904}{3889}, \frac{563}{3889}, \frac{692}{3889}, \frac{281}{3889}, \frac{389}{3889}, \frac{40}{3889}, \frac{343}{3889},\right.$$
$$\left.\frac{111}{3889}, \frac{57}{3889}\right], \left[\frac{605}{4989}, \frac{362}{4989}, \frac{12}{1663}, \frac{29}{1663}, \frac{971}{4989}, \frac{716}{4989}, \frac{266}{1663}, \frac{146}{4989}, \frac{851}{4989},\right.$$
$$\left.\frac{139}{1663}\right], \left[\frac{103}{2186}, \frac{287}{2186}, \frac{127}{4372}, \frac{269}{2186}, \frac{257}{4372}, \frac{473}{2186}, \frac{137}{1093}, \frac{138}{1093}, \frac{621}{4372}, \frac{3}{4372}\right],$$
$$\left.\left[\frac{316}{2197}, \frac{271}{4394}, \frac{36}{2197}, \frac{601}{4394}, \frac{959}{4394}, \frac{42}{2197}, \frac{213}{4394}, \frac{59}{338}, \frac{469}{4394}, \frac{163}{2197}\right]\right]$$

\> *StSa*(*MarkovP*, 4000)

[0.1002500000, 0.1075000000, 0.07300000000, 0.1157500000, 0.06975000000, 0.1260000000, 0.09450000000, 0.08750000000, 0.1230000000, 0.1027500000] **(7)**

```
> StSp(MarkovP, 4000)
  #Maple is not running StSp for some reason
> evalf(StS(MarkovP))
```

[0.1005005112, 0.1102931620, 0.08103080713, 0.1159156158, 0.06824177820, 0.1238845481, 0.08700742089, 0.08379589819, 0.1273375444, 0.1019927141] **(8)**

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>
> #2.
  #a.
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#The recursion works because we can base the recurrence on the first round. After the first round, the player will have either lost, and lost one dollar from i, or have won, and gained one dollar from i. Since the probability of winning and losing is the same with a coin, .5, we can multiply the possible results by .5. If you start with 0 dollars, and the odds of winning and losing are even, you have more of a probability of losing the whol game because you have started off at a defecit, however, if you have started off with the maximum amount of money already, you have already essentially won.

#b.

$$\# \frac{i}{N} = \frac{1}{2}\left(\frac{i-1}{N} + \frac{i+1}{N}\right)$$

$$\# \frac{i}{N} = \frac{1}{2}\left(\frac{2\,i}{N}\right)$$

$$\# \frac{i}{N} = \frac{i}{N}$$

$$\#yN(0) = \frac{0}{N} = 0$$

$$\# \, yN(N) = \frac{N}{N} = 1$$

#c.

#The recurrence works because given i dollars at first the expected number of turns to the game's end is E(i). Say the at the current turn, the player loses a dollar. The next expected number of turns is E(i-1) +1, to indicate that a turn has already been made. The same is true for winning the round, except it would be E(i+1)+1. Again the probabilities of winning and losing are the same with a coin, so the probabilities are added, multiplied by .5, and the 1 can be added to the total product.

#d,

$$\#i(N\text{-}i) = \frac{1}{2}\left(\left(i-1\right)\left(N-\left(i-1\right)\right) + \left(i+1\right)\left(N-\left(i+1\right)\right)\right) + 1$$

$$\#i(N\text{-}i) = \frac{1}{2}\left(Ni - i^2 + i - N + i - 1 + Ni - i^2 - i + N - i - 1\right) + 1$$

$\#i(N\text{-}i)=\frac{1}{2}\left(2\,Ni-2\,i^2-2\right)+1$

$\#i(N\text{-}i)=Ni\text{-}i^2-1+1$

$\#i(N\text{-}i)=i(N\text{-}i)$

$\#EN(0)=0(N\text{-}0)=0$

> $\#EN(N)=N(N\text{-}N)=N(0)=0$