Homework-7

2) a) $x_N(i)$ - prob of exiting a winner

fair coin $\therefore$ $x_N(i) = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_N(i-1) \\ x_N(i+1) \end{bmatrix}$

$x_N(i)$

0.5 0.5 0.5 0.5 $\therefore$ $x_N(i) = \frac{1}{2}[x_N(i-1) + x_N(i+1)]$

$x_N(i-1)$ $(x_N(i+1))$,

[As we have a fair coin probability of winning with i-1 is same as prob of losing with i+1]

Boundary conditions $x_N(0) = 0$

This is true as a broke gambler will never win.

$x_N(N) = 1$ this is true because Gambler ~~with~~ ~~cost~~ \$ is considered a winner when he has max capital and with this we will exit the game (never play)

b) To prove $y_N(i) = \frac{i}{N}$ satisfies the recurrence

$$\frac{1}{2}\left[\frac{i-1}{N} + \frac{i+1}{N}\right] = \frac{1}{2}\left[2\frac{i}{N}\right] = \frac{i}{N} = \text{LHS}$$

Hence, proved.

Explanation I think $\frac{i}{N}$ is a explicit formula as the larger the amt i put in the game

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the closer you are to N and hence a larger prob of winning than going broke.

c) $E_N(i)$ - expected number of rounds to complete a gambler's ruin.

$$E_N(i) = \frac{1}{2}(E_n(i-1) + E_N(i+1)) + 1$$

$E_n(i-1)$ and $E_n(i+1)$ are the expected number of rounds to complete the game after losing or winning a dollar in this game. We divide by half as the game will go in either 1 direction and (add) include 1 as we have to count the current game.

$E_N(0) = 0 \quad E_N(N) = 0$

This is cos at 0 the gambler is broke & at N has won. In either situation the game is already over.

d) $Z_N(i) = i(N-i)$

$$\frac{1}{2}[(i-1)(N-i-1) + (i+1)(N-i+1)) + 1$$

$$= \frac{1}{2}[iN - i^2 - \cancel{i} - \cancel{N} + \cancel{i} + 1 + in - i^2 + \cancel{i} + \cancel{N} - \cancel{i} + 1] + 1$$

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$= \frac{1}{2}[2iN - 2i^2 + 2] + 1$

$= iN - i^2 + 2$

$= i(N-i)$

Exp: Not sure

4) for i from 1 to 10 do
   P:= RandSM(10):
   StSa(P, 4000)
   StSp(P, 4000
   StS(P)
   Od:

read "C:/Users/rmn74/Documents/M7.txt"
Help7( )

GR(p,i,N), GRt(p,i,N), GRm(N,p), OneStepMarkov(P,i), MarkovTrip(P,K), StSa(P,K) , StS(P), StSp(P,K), RandSM(N) **(1)**

#1 pr is the number of games won and pr\k give prob of exiting a winner, t\k is the avg time for a game to end

$EstGR :=$ **proc**(p, i, N, k) **local** pr, t, x, L, a :
$pr := 0$:
$t := 0$:
$L :+ [1]$:
for a **from** 1 **to** k **do**
$x := GRt(p, i, N)$:
if $x[1] = 1$ **then** $pr := pr + 1$: **fi**:
$t := t + x[2]$:
od:
$L := \left[\frac{pr}{k}, \frac{t}{k}\right]$:
$RETURN(L)$:
end:

#3
$ExactFairGR :=$**proc**(i, N) **local** x, En :
$x := \frac{i}{N}$:
$En := i(N - i)$:
$RETURN(x, En)$:
end:

$ESTp := [\ ]$:
$ESTt := [\ ]$:
$EXACTp := [\ ]$:
$EXACTt := [\ ]$:
for a **from** 1 **to** 19 **do**
$L := EstGR\left(\frac{1}{2}, a, 20, 3000\right)$:
$M := ExactFairGR(a, 20)$:
$ESTp[a] := L[1]$:
$ESTt := L[2]$:
$EXACTp := M[1]$:
$EXACTt := M[2]$:
od:

Error, out of bound assignment to a list