```
> #OKay to post
> #Anusha Nagar, Homework 7, 9.27.2021
> read "C:/Users/an646/Documents/M5.txt";
> read "C:/Users/an646/Documents/M7.txt";
> Help7( )
```

GR(p,i,N), GRt(p,i,N), GRm(N,p), OneStepMarkov(P,i), MarkovTrip(P,K), StSa(P,K) , StS(P), StSp(P,K), RandSM(N) **(1)**

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> #Problem 1
> EstGRt := proc(p, i, N, k) local X, x, d, c :
  X := RandomVariable(Bernoulli(p)) :
  c := 0 :
  x := i :
  while k > 0 do
   while x > 0 and x < N do
    c := c + 1 :
    d := trunc(Sample(X, 1)[1]) :
     if d = 0 then
      x := x - 1 :
      else
     x := x + 1 :
     fi:
    od:
    if x = 0 then
     RETURN([0, c]) :
     elif x = N then
     RETURN([1, c]) :
     fi:
   k := k - 1;
   end:
  end:
> #Problem2
> #Problem 2a
```

> #It is easy to see why the boundary conditions are true. If we exit with 0 dollars, we lose (probabiliy of exiting a winner is 0), and if we exit with N (max capital), then we win (probability of exiting a winner is 1)

> #As for the recurrence, the probabilitiy of exiting a winner with i dollars is the average of exiting a winner with 1 less dollar and 1 more dollar. This can also be thought of as the average of wins and losses, or the average of the probability that you leave with one more dollar (after winning one more round) and the probability that you leave with one less dollar (that you lose the round). We know that with a fair coin, we have equal probabilitiy of winning a round or losing a round, so p=0.5. We build the recurrence by seeing that the probability that we either win or lose the round is 0.5 -> we let XN(i+1) be the probability that we win the game while winning the first round, and XN(i-1) be the probability that we win even after losing the first round. Summing together, we see that XN(i) = 0.5·XN(i+1)+0.5·XN(i-1), which is what we have

> #I'd also like to note that I referenced the following websites to learn more about Gambler's Ruin, as I was a little confused after class: https://www.probabilitycourse.com/chapter14/Chapter_14.pdf and https://web.mit.edu/neboat/Public/6.042/randomwalks.

pdf

> #Problem 2b

> #Once again, it's easy to see that the boundary conditions satisfy the closed form expression. When i = 0, xN = 0, and when i = N, xN = 1.

> #We see that the closed form of the expression is accurate by taking yN(i)-i over N, yN(i-1) as (i-1) over N, and yN(i+1) as (i+1) over N. Plugging in, we see:

> #xN(i)=0.5[(i-1)overN+(i+1)overN]. Simplifying, we get that xN(i)=i over N as well.

> #Problem 2c

> #The boundary conditions are easy to see, as if we come in with either 0 dollars or N dollars, we have immediately won or lost after 0 rounds.

> #I referenced the following website for this question: https://web.mit.edu/neboat/Public/6.042/randomwalks.pdf . Unfortunately, I was still confused about the expected number of rounds, and how to prove this recurrence.

> #Problem 2d

> #It is easy to see that the boundary conditions are satisfied, as when i = 0, En = 0, and when i = N, En = 0.

> #We let zn(i)=i(N-i). Therefore, Zn(i-1)=(i-1)(N-i+1)=Ni-$i^2 + i - N + i - 1$, **and** $Zn(i+1) = (i+1)(N-i-1) = Ni - i^2 - i + N - i - 1$. If we plug this into the $En(i)$ formula, we get $En(i) = 0.5[(i-1)(N-i+1) + (i+1)(N-i-1) + 1]$ which equals $En(i) = i(N-i)$

>

> #Problem 3

>

```
> ExactFairGR := proc(i, N) local x, e :
x := i/N :
e := i*(N-i) :
print([x, e])
end:
```

> ExactFairGR(1, 20)

$$\left[\frac{1}{20}, 19\right] \quad (2)$$

> ExactFairGR(2, 20)

$$\left[\frac{1}{10}, 36\right] \quad (3)$$

> ExactFairGR(3, 20)

$$\left[\frac{3}{20}, 51\right] \quad (4)$$

> ExactFairGR(4, 20)

$$\left[\frac{1}{5}, 64\right] \quad (5)$$

> ExactFairGR(5, 20)

$$\left[\frac{1}{4}, 75\right] \quad (6)$$

> ExactFairGR(6, 20)

$$\left[\frac{3}{10}, 84\right] \quad (7)$$

> *ExactFairGR*(7, 20)

$$\left[\frac{7}{20}, 91\right] \quad (8)$$

> *ExactFairGR*(8, 20)

$$\left[\frac{2}{5}, 96\right] \quad (9)$$

> *ExactFairGR*(9, 20)

$$\left[\frac{9}{20}, 99\right] \quad (10)$$

> *ExactFairGR*(10, 20)

$$\left[\frac{1}{2}, 100\right] \quad (11)$$

> *ExactFairGR*(11, 20)

$$\left[\frac{11}{20}, 99\right] \quad (12)$$

> *ExactFairGR*(12, 20)

$$\left[\frac{3}{5}, 96\right] \quad (13)$$

> *ExactFairGR*(13, 20)

$$\left[\frac{13}{20}, 91\right] \quad (14)$$

> *ExactFairGR*(14, 20)

$$\left[\frac{7}{10}, 84\right] \quad (15)$$

> *ExactFairGR*(15, 20)

$$\left[\frac{3}{4}, 75\right] \quad (16)$$

> *ExactFairGR*(16, 20)

$$\left[\frac{4}{5}, 64\right] \quad (17)$$

> *ExactFairGR*(17, 20)

$$\left[\frac{17}{20}, 51\right] \quad (18)$$

> *ExactFairGR*(18, 20)

$$\left[\frac{9}{10}, 36\right] \quad (19)$$

> *ExactFairGR*(19, 20)

$$\left[\frac{19}{20}, 19\right] \quad (20)$$

> *EstGRt*(0.5, 1, 20, 3000)

$$[0, 1] \quad (21)$$

> *EstGRt*(0.5, 2, 20, 3000)

$$[0, 4] \quad (22)$$

> *EstGRt*(0.5, 3, 20, 3000)

(23)

$$[0, 15] \quad (23)$$

> $EstGRt(0.5, 4, 20, 3000)$

$$[0, 6] \quad (24)$$

> $EstGRt(0.5, 5, 20, 3000)$

$$[0, 19] \quad (25)$$

> $EstGRt(0.5, 6, 20, 3000)$

$$[1, 98] \quad (26)$$

> $EstGRt(0.5, 7, 20, 3000)$

$$[1, 263] \quad (27)$$

> $EstGRt(0.5, 8, 20, 3000)$

$$[0, 86] \quad (28)$$

> $EstGRt(0.5, 9, 20, 3000)$

$$[0, 109] \quad (29)$$

> $EstGRt(0.5, 10, 20, 3000)$

$$[1, 124] \quad (30)$$

> $EstGRt(0.5, 11, 20, 3000)$

$$[1, 125] \quad (31)$$

> $EstGRt(0.5, 12, 20, 3000)$

$$[1, 16] \quad (32)$$

> $EstGRt(0.5, 13, 20, 3000)$

$$[1, 45] \quad (33)$$

> $EstGRt(0.5, 14, 20, 3000)$

$$[0, 126] \quad (34)$$

> $EstGRt(0.5, 15, 20, 3000)$

$$[0, 223] \quad (35)$$

> $EstGRt(0.5, 16, 20, 3000)$

$$[0, 60] \quad (36)$$

> $EstGRt(0.5, 17, 20, 3000)$

$$[1, 13] \quad (37)$$

> $EstGRt(0.5, 18, 20, 3000)$

$$[1, 2] \quad (38)$$

> $EstGRt(0.5, 19, 20, 3000)$

$$[1, 15] \quad (39)$$

>

> #Problem 4

> $P := RandSM(10)$

$$P := \left[\left[\frac{481}{2683}, \frac{129}{2683}, \frac{126}{2683}, \frac{875}{5366}, \frac{67}{5366}, \frac{214}{2683}, \frac{98}{2683}, \frac{915}{5366}, \frac{365}{2683}, \frac{683}{5366}\right], \left[\frac{41}{194}, \frac{9}{2231}, \frac{439}{2231}, \frac{389}{2231}, \frac{37}{4462}, \frac{53}{4462}, \frac{358}{2231}, \frac{428}{2231}, \frac{35}{2231}, \frac{113}{4462}\right], \left[\frac{290}{2979}, \frac{965}{5958}, \frac{11}{331}, \frac{247}{1986}, \frac{118}{2979}, \frac{767}{5958}, \frac{269}{2979}, \frac{304}{2979}, \frac{623}{5958}, \frac{39}{331}\right], \left[\frac{945}{5416}, \frac{735}{5416}, \frac{42}{677}, \frac{893}{5416}, \frac{1}{677}, \frac{147}{1354}, \frac{377}{5416}, \frac{385}{5416}, \frac{137}{2708}, \frac{875}{5416}\right], \left[\frac{62}{2307}, \frac{229}{2307}, \frac{469}{4614}, \frac{443}{2307}, \right. \quad (40)$$

$\frac{211}{4614}, \frac{229}{1538}, \frac{34}{2307}, \frac{479}{2307}, \frac{13}{2307}, \frac{727}{4614}\Big], \Big[\frac{246}{1625}, \frac{179}{975}, \frac{46}{4875}, \frac{97}{4875}, \frac{19}{125},$
$\frac{19}{4875}, \frac{183}{1625}, \frac{27}{1625}, \frac{178}{975}, \frac{21}{125}\Big], \Big[\frac{67}{6620}, \frac{611}{6620}, \frac{461}{3310}, \frac{237}{3310}, \frac{49}{331}, \frac{353}{3310}, \frac{40}{331},$
$\frac{299}{3310}, \frac{999}{6620}, \frac{463}{6620}\Big], \Big[\frac{223}{1568}, \frac{11}{112}, \frac{607}{6272}, \frac{477}{6272}, \frac{461}{3136}, \frac{129}{1568}, \frac{5}{128}, \frac{177}{1568},$
$\frac{341}{6272}, \frac{237}{1568}\Big], \Big[\frac{354}{4541}, \frac{782}{4541}, \frac{537}{4541}, \frac{843}{4541}, \frac{110}{4541}, \frac{31}{4541}, \frac{808}{4541}, \frac{5}{4541}, \frac{809}{4541},$
$\frac{262}{4541}\Big], \Big[\frac{393}{5176}, \frac{213}{2588}, \frac{361}{2588}, \frac{363}{2588}, \frac{409}{5176}, \frac{599}{5176}, \frac{579}{5176}, \frac{371}{5176}, \frac{573}{5176}, \frac{189}{2588}\Big]\Big]$

> *StSa*(*P*, 4000)

[0.1267500000, 0.1102500000, 0.09600000000, 0.1360000000, 0.05525000000, 0.08575000000, 0.09125000000, 0.1002500000, 0.08875000000, 0.1097500000] **(41)**

> *StSp*(*P*, 4000)

> *#Maple could not compute the line above*

> *StS*(*P*)

$\Big[\frac{108972201199673658497051037869109 6158}{895835293272322667184402930747297 2521},$
$\frac{941536191187484639020372839233714108}{8958352932723226671844029307472972521},$
$\frac{849126248018418112888767069283338960}{8958352932723226671844029307472972521},$
$\frac{170915295946650893348417926603906008}{1279764704674746667406289901067567503},$
$\frac{547607844144060946003655719418582070}{8958352932723226671844029307472972521},$
$\frac{706407097619873047776242560390573125}{8958352932723226671844029307472972521},$
$\frac{843964050949431234433770189154394920}{8958352932723226671844029307472972521},$
$\frac{130616709832891959773276158950645632}{1279764704674746667406289901067567503},$
$\frac{888306149666261711286405107114714844}{8958352932723226671844029307472972521},$
$\frac{980959298684160423612446845304696856}{8958352932723226671844029307472972521}\Big]$ **(42)**

> *evalf*(%)

[0.1216431212, 0.1051014844, 0.09478597845, 0.1335521251, 0.06112818375, 0.07885457326, 0.09420973446, 0.1020630662, 0.09915953930, 0.1095021938] **(43)**

> *#In comparing, we see that the values are fairly close between StSa and StS of the matrix with the given K value.*