

```

> read("M7.txt")
> Help7( )
GR(p,i,N), GRt(p,i,N), GRm(N,p), OneStepMarkov(P,i), MarkovTrip(P,K), StSa(P,K) , StS(P), (1)
  StSp(P,K), RandSM(N)
1)
> GRt(0.45, 5, 10)[1]
0 (2)
> EstGR := proc(p, i, N, K) local times_won, t, prob, duration, avr_duration :
  times_won := (add(GRt(p, i, N) [1], t = 1 ..K));
  prob := evalf( $\frac{\text{times\_won}}{K}$ );
  duration := (add(GRt(p, i, N) [2], t = 1 ..K));
  avr_duration := evalf( $\frac{\text{duration}}{K}$ );
  return ([prob, %]);
end:
> EstGR(0.45, 5, 10, 1000)
[0.2830000000, 23.24000000] (3)
> #The probability of exiting a winner (from this specific simulation) is 0.283 and the average
  duration of the game is 23.240 rounds.
3)
> ExactFairGR := proc(i, N) local prob_winner, avr_duration :
  prob_winner := evalf( $\frac{i}{N}$ );
  avr_duration := evalf(i · (N - i));
  return ([prob_winner, avr_duration])
end
ExactFairGR := proc(i, N) (4)
  local prob_winner, avr_duration;
  prob_winner := evalf(i / N);
  avr_duration := evalf(i * (N - i));
  return [prob_winner, avr_duration]
end proc
> ExactFairGR(1, 20)
[0.05000000000, 19.] (5)
> EstGR(0.5, 1, 20, 3000)
[0.04500000000, 19.36066667] (6)
>
> ExactFairGR(2, 20)
[0.1000000000, 36.] (7)
> EstGR(0.5, 2, 20, 3000)
[0.09100000000, 37.01466667] (8)

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> #not sure if the homework directions mean to run each function 19 times? From these three individual trials it looks like the estimation is quite close to the exact numbers. Next, I'll sum $i=1..19$ for both functions in order to summarize how close the estimation predicts the actual.

> add(ExactFairGR(i, 20), i = 1 ..19)
[9.500000000, 1330.] (9)

> add(EstGR(0.5, i, 20, 3000), i = 1 ..19)
[9.512333333, 1325.720001] (10)

> #This took more than 10 min to actually run

4)

> RandSM(10)
[[[93, 173, 243, 471, 10, 9, 20, 93, 43, 219], [131, 1351, 1351, 2702, 1351, 1351, 1351, 193, 1351, 2702, 1351], [4434, 113, 569, 277, 139, 23, 467, 326, 86, 30], [111, 421, 1478, 4434, 2217, 739, 1478, 2217, 2217, 2217, 739], [724, 2896, 107, 233, 707, 419, 103, 611, 383, 15], [1, 193, 1, 1448, 1448, 5792, 2896, 5792, 5792, 5792, 1448], [11, 2596, 5192, 161, 79, 373, 239, 309, 2, 965], [695, 161, 371, 209, 5192, 1298, 5192, 1298, 2596, 11, 5192], [4716, 2358, 4716, 4716, 85, 122, 445, 59, 71, 371], [195, 86, 159, 383, 137, 1572, 1179, 2358, 524, 1572, 2358], [3983, 569, 3983, 3983, 3983, 31, 954, 212, 508, 802], [137, 295, 356, 479, 287, 943, 3983, 3983, 3983, 3983, 3983], [1857, 1857, 5571, 5571, 1857, 5571, 74, 269, 127, 304], [131, 243, 799, 313, 499, 447, 407, 5571, 5571, 1857, 1857], [736, 2576, 5152, 2576, 5152, 5152, 5152, 393, 51, 221], [67, 274, 863, 312, 952, 311, 966, 135, 5152, 736, 5152], [1151, 5755, 5755, 5755, 5755, 5755, 5755, 1151, 459, 608], [421, 948, 138, 718, 387, 824, 751, 2, 2, 5755, 5755], [5885, 5885, 1177, 5885, 5885, 5885, 5885, 55, 535, 182, 1177]]]

> P := %
P := [[[93, 173, 243, 471, 10, 9, 20, 93, 43, 219], [131, 1351, 1351, 2702, 1351, 1351, 1351, 193, 1351, 2702, 1351], [4434, 113, 569, 277, 139, 23, 467, 326, 86, 30], [111, 421, 1478, 4434, 2217, 739, 1478, 2217, 2217, 2217, 739], [724, 2896, 107, 233, 707, 419, 103, 611, 383, 15], [1, 193, 1, 1448, 1448, 5792, 2896, 5792, 5792, 5792, 1448], [11, 2596, 5192, 161, 79, 373, 239, 309, 2, 965], [695, 161, 371, 209, 5192, 1298, 5192, 1298, 2596, 11, 5192], [4716, 2358, 4716, 4716, 85, 122, 445, 59, 71, 371], [195, 86, 159, 383, 137, 1572, 1179, 2358, 524, 1572, 2358], [3983, 569, 3983, 3983, 3983, 31, 954, 212, 508, 802], [137, 295, 356, 479, 287, 943, 3983, 3983, 3983, 3983, 3983], [1857, 1857, 5571, 5571, 1857, 5571, 74, 269, 127, 304], [131, 243, 799, 313, 499, 447, 407, 5571, 5571, 1857, 1857], [736, 2576, 5152, 2576, 5152, 5152, 5152, 393, 51, 221], [67, 274, 863, 312, 952, 311, 966, 135, 5152, 736, 5152], [1151, 5755, 5755, 5755, 5755, 5755, 5755, 1151, 459, 608], [421, 948, 138, 718, 387, 824, 751, 2, 2, 5755, 5755], [5885, 5885, 1177, 5885, 5885, 5885, 5885, 55, 535, 182, 1177]]] (12)

$$\left[\frac{31}{3983}, \frac{954}{3983}, \frac{212}{3983}, \frac{508}{3983}, \frac{802}{3983} \right], \left[\frac{137}{1857}, \frac{295}{1857}, \frac{356}{5571}, \frac{479}{5571}, \frac{287}{1857}, \frac{943}{5571}, \right.$$

$$\frac{74}{5571}, \frac{269}{5571}, \frac{127}{1857}, \frac{304}{1857} \left. \right], \left[\frac{131}{736}, \frac{243}{2576}, \frac{799}{5152}, \frac{313}{2576}, \frac{499}{5152}, \frac{447}{5152}, \frac{407}{5152}, \right.$$

$$\frac{393}{5152}, \frac{51}{736}, \frac{221}{5152} \left. \right], \left[\frac{67}{1151}, \frac{274}{5755}, \frac{863}{5755}, \frac{312}{5755}, \frac{952}{5755}, \frac{311}{5755}, \frac{966}{5755}, \frac{135}{1151}, \right.$$

$$\frac{459}{5755}, \frac{608}{5755} \left. \right], \left[\frac{421}{5885}, \frac{948}{5885}, \frac{138}{1177}, \frac{718}{5885}, \frac{387}{5885}, \frac{824}{5885}, \frac{751}{5885}, \frac{2}{55}, \frac{2}{535}, \right.$$

$$\left. \frac{182}{1177} \right]$$

> *StSa*(*P*, 4000)
 [0.09575000000, 0.1155000000, 0.08825000000, 0.1195000000, 0.09075000000, (13)
 0.08325000000, 0.1337500000, 0.08525000000, 0.06500000000, 0.1230000000]

> *StSp*(*P*, 4000) [Length of output exceeds limit of 1000000] (14)

> *evalf*(*StS*(*P*)) (15)
 [0.08988526553, 0.1133937420, 0.08693393162, 0.1164119037, 0.09610443548,
 0.08493029740, 0.1308676696, 0.08682142153, 0.06884349105, 0.1258078421]

>