HW7:

2a) $X_N(i) = \frac{1}{2}\left(X_N(i-1) + X_N(i+1)\right)$

- The probability is modeled by this linear recurrence b/c entering capital i will either have 1 subtracted or added after each trial w/ probability 0.5 (modeled by the $\frac{1}{2}$ multiplying the equation) b/c a fair coin has prob. 0.5 of landing on heads or tails.
- $X_N(0) = 0$ must be true b/c if starting capital is 0 it is impossible for a player to even win.
- $X_N(N) = 1$ is true b/c if starting capital is N, then the max capital has already been reached and the player has already won.

b) $y_N(i) = \frac{i}{N}$

$$\frac{i}{N} = \frac{1}{2}\left(\frac{i}{N}(i-1) + \frac{i}{N}(i+1)\right)$$

$$\frac{i}{N} = \frac{1}{2}\left(\frac{i^2}{N} - \frac{i}{N} + \frac{i^2}{N} + \frac{i}{N}\right)$$

$$\frac{i}{N} = \frac{2i^2}{2N} = \frac{i}{N} \checkmark$$

$$y_N(0) = \frac{1}{2}\left(\frac{0}{N}(0-1) + \frac{0}{N}(1)\right) = \frac{1}{2}(0)$$
$$0 = 0 \checkmark$$

$$y_N(N) = \frac{1}{2}\left(\frac{N}{N} \cdot \frac{N}{N}(N-1) + \frac{N}{N}(N+1)\right)$$
$$1 = \frac{2N}{2}$$
$$1 = 1 \checkmark$$

- This explicit formula works because the entering capital divided by the max amount w/ prob. 0.5 of winning/losing will represent the prob. of exiting the game as a winner.

c) $E_N(i) = \frac{1}{2}(E_N(i-1) + E_N(i+1)) + 1$

- This linear recursion is the expected # of rounds to complete the game b/c by adding 1 the equation predicts the amount of rounds before loss/win.

$E_N(0) = 0$ => if $i = 0$, then player already lost (has no money and is ruined). It will take 0 additional rounds to exit.

$E_N(N) = 0$ => if $i = N$ the player already won, they already have the max capital so it will take 0 additional rounds.

d) $Z_N(i) = i(N-i)$

$= \frac{1}{2}\left(i(N-1)(i-1) + i(N-i)(i+1)\right) + 1$

$= \frac{1}{2}\left(i(Ni - N - i + 1) + i(Ni + N - i^2 - i)\right) + 1$

$= \left(\cancel{N}i^2 - \cancel{N}i - i^2 + i + \cancel{N}i^2 + \cancel{N}i - i^3 - i^2\right) + 1$

$= 2Ni^2 - i^3 - 2i^2 + i + 1$

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