Max Mekhanikov - HW6 - ok to post

1)

$n_0(t) = 0.1 n_0(t-1) + 1.2 n_1(t-1) + 0.9 n_2(t-1) + 0.1 n_3(t-1)$

$n_1(t) = 0.95 n_0(t-1)$

$n_2(t) = 0.97 n_1(t-1)$

$n_3(t) = 0.9 n_2(t-1)$

$n_1(t-1) = 0.95 n_0(t-2)$

$n_2(t-1) = 0.97 n_1(t-2)$, $n_1(t-2) = 0.95 n_0(t-3)$

$n_2(t-1) = 0.97(0.95) n_0(t-3)$

$n_3(t-1) = 0.9 n_2(t-2)$, $n_2(t-2) =$

$0.97(0.95) n_0(t-4)$

$n_3(t-1) = 0.9(0.97(0.95(n_0(t-4))))$

$n_0(t) = 0.1 n_0(t-1) + 1.2(0.95) n_0(t-2) + 0.9(0.9215) n_0(t-3)$

$+ 0.1(0.82935) n_0(t-4)$

$n_0(t) = 0.1 n_0(t-1) + 1.14 n_0(t-2) + 0.82935 n_0(t-3) + 0.082935 n_0(t-4)$

$REQ = [0.1, 1.14, 0.82935, 0.082935]$

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> # Max Mekhanikov - Homework 6 - Okay to post
>
  # Question 1
> #GrowthCe(REC): The  EXACT  growth constant of the recurrence REC using the characteristic equation
  GrowthCe :=proc(REC) local x, i :
  evalf([solve(1-add(REC[i]/x^i, i=1..nops(REC)))])[1]:

  end:
>
> REC := [0.1, 1.14, 0.82935, 0.082935]:
> GrowthCe(REC)
```

$$1.385732629$$

2) Leslie Matrix

$$\vec{N}(t) = \begin{bmatrix} n_0(t) \\ n_1(t) \\ n_2(t) \\ n_3(t) \end{bmatrix} = \begin{bmatrix} 0.1 & 1.2 & 0.9 & 0.1 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.97 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \end{bmatrix} \begin{bmatrix} n_0(t-1) \\ n_1(t-1) \\ n_2(t-1) \\ n_3(t-1) \end{bmatrix}$$

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# Question 2
with(LinearAlgebra) :
Leslie := Matrix([[0.1, 1.2, 0.9, 0.1], [0.95, 0, 0, 0], [0, 0.97, 0, 0], [0, 0, 0.9, 0]])
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$$Leslie := \begin{bmatrix} 0.1 & 1.2 & 0.9 & 0.1 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.97 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \end{bmatrix}$$

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Eigenvalues(Leslie)
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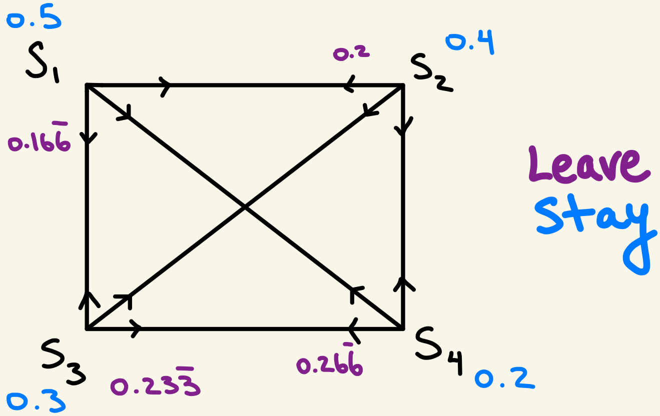
$$\begin{bmatrix} 1.38573262885364 + 0.\,I \\ -0.583351516086360 + 0.403125877485025\,I \\ -0.583351516086360 - 0.403125877485025\,I \\ -0.119029596680917 + 0.\,I \end{bmatrix}$$

Largest eigenvalue = GrowthCe ✅

3)



$$\begin{bmatrix}S_1\\S_2\\S_3\\S_4\end{bmatrix}(t)=\underbrace{\begin{bmatrix}0.5 & \frac{0.5}{3} & \frac{0.5}{3} & \frac{0.5}{3}\\ 0.2 & 0.4 & 0.2 & 0.2\\ \frac{0.7}{3} & \frac{0.7}{3} & 0.3 & \frac{0.7}{3}\\ \frac{0.8}{3} & \frac{0.8}{3} & \frac{0.8}{3} & 0.2\end{bmatrix}}_{P}\begin{bmatrix}S_1(t-1)\\S_2(t-1)\\S_3(t-1)\\S_4(t-1)\end{bmatrix}$$

> # Question 3

>

$$P := Matrix\left(\left[\left[0.5, \frac{0.5}{3}, \frac{0.5}{3}, \frac{0.5}{3}\right], [0.2, 0.4, 0.2, 0.2], \left[\frac{0.7}{3}, \frac{0.7}{3}, 0.3, \frac{0.7}{3}\right], \left[\frac{0.8}{3}, \frac{0.8}{3}, \frac{0.8}{3}, 0.2\right]\right]\right)$$

$$P := \begin{bmatrix} 0.5 & 0.1666666667 & 0.1666666667 & 0.1666666667\\ 0.2 & 0.4 & 0.2 & 0.2\\ 0.2333333333 & 0.2333333333 & 0.3 & 0.2333333333\\ 0.2666666667 & 0.2666666667 & 0.2666666667 & 0.2 \end{bmatrix}$$

> $evalm(P^{1000})$

$$\begin{bmatrix} 0.315197007189532 & 0.262664172672962 & 0.225140719443162 & 0.196998129500332\\ 0.315197007144515 & 0.262664172635447 & 0.225140719411007 & 0.196998129472196\\ 0.315197007112359 & 0.262664172608651 & 0.225140719388038 & 0.196998129452099\\ 0.315197007176892 & 0.262664172662428 & 0.225140719434133 & 0.196998129492432 \end{bmatrix}$$

All rows are equivalent and S1 is the most popular, followed by S2, S3, and S4 in that order.