OK to Post!

Anusha Nagar, hw 6, 9.27.2021

(1) Survival Rate: $0 \to 1$: 0.95

$1 \to 2$: 0.97

$2 \to 3$: 0.9

Fertility Rate: 0: 0.1

1: 1.2

2: 0.9

3: 0.1

$$\begin{cases} n_0(t) = 0.1\,n_0(t-1) + 1.2\,n_1(t-1) + 0.9\,n_2(t-1) + 0.1\,n_3(t-1) \\ n_1(t) = 0.95\,n_0(t-1) \\ n_2(t) = 0.97\,n_1(t-1) \\ n_3(t) = 0.9\,n_2(t-1) \end{cases}$$

$n_0(t-1)$ ★

$n_1(t-1) = 0.95\,n_0(t-2)$ ★

$n_2(t-1) = 0.97\,n_1(t-2)$

↳ $n_1(t-2) = 0.95\,n_0(t-3)$

↳ $n_2(t-1) = 0.97(0.95\,n_0(t-3))$ ★ $\Rightarrow = 0.9215\,n_0(t-3)$

$n_3(t-1) = 0.9\,n_2(t-2)$

↳ $n_2(t-2) = 0.97\,(0.95\,n_0(t-4))$

$n_3(t-1) = 0.9(0.97\,(0.95\,n_0(t-4)))$ ★ $\Rightarrow 0.82935\,n_0(t-4)$

Then:

$$n_0(t) = 0.1\,n_0(t-1) + 1.2\,n_1(t-1) + 0.9\,n_2(t-1) + 0.1\,n_3(t-1)$$

$$= 0.1\,n_0(t-1) + 1.2(0.95)\,n_0(t-2) + 0.9(0.9215)\,n_0(t-3) + 0.1(0.82935)\,n_0(t-4)$$

$$n_0(t) = 0.1\,n_0(t-1) + 1.14\,n_0(t-2) + 0.82935\,n_0(t-3) + 0.082935\,n_0(t-4)$$

REC = [0.1, 1.14, 0.82935, 0.082935]

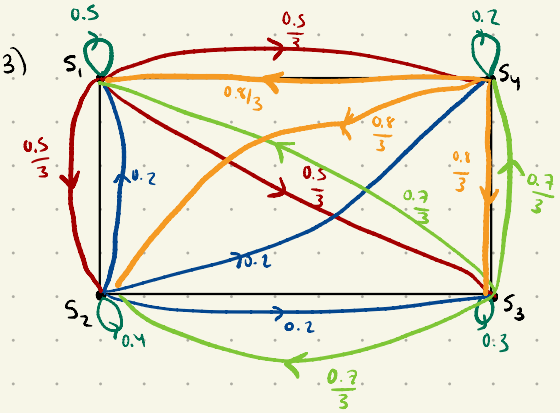
Maple is attached. Both methods have the same growth constant

(2) Leslie Matrix

$$\begin{bmatrix} 0.1 & 1.2 & 0.9 & 0.1 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.97 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \end{bmatrix}$$

Eigenvalues done in Maple (attached). It agrees with Problem 1

(3)



$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} = \begin{bmatrix} 0.5 & \frac{0.5}{3} & \frac{0.5}{3} & \frac{0.5}{3} \\ 0.2 & 0.4 & 0.2 & 0.2 \\ \frac{0.7}{3} & \frac{0.7}{3} & 0.3 & \frac{0.7}{3} \\ \frac{0.8}{3} & \frac{0.8}{3} & \frac{0.8}{3} & 0.2 \end{bmatrix} \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} (t-1)$$

(a) $P = \begin{bmatrix} 0.5 & \frac{0.5}{3} & \frac{0.5}{3} & \frac{0.5}{3} \\ 0.2 & 0.4 & 0.2 & 0.2 \\ \frac{0.7}{3} & \frac{0.7}{3} & 0.3 & \frac{0.7}{3} \\ \frac{0.8}{3} & \frac{0.8}{3} & \frac{0.8}{3} & 0.2 \end{bmatrix}$

(b) Maple $\Rightarrow$ see attached

```
> #OK to post
> #Anusha Nagar, 9.27.2021, Homework 6
>
> #Problem 1
> read "C:/Users/an646/Documents/M5.txt" :
> read "C:/Users/an646/Documents/M7.txt" :
> Help5( )
```

RecToSeq(INI,REC,N), GrowthC(INI,REC,K) , GrowthCe(REC)
LeslieMod(SUR,FER): e.g. LeslieMod([9/10,9/10],[0,1,1]);
LeslieMat(SUR,FER); e.g. LeslieMat([9/10,9/10],[0,1,1]); **(1)**

```
> Help7( )
```

GR(p,i,N), GRt(p,i,N), GRm(N,p), OneStepMarkov(P,i), MarkovTrip(P,K), StSa(P,K) , StS(P), StSp(P,K), RandSM(N) **(2)**

```
>
>
> GrowthCe([0.1, 1.14, 0.82935, 0.082935])
```

$$1.385732629 \tag{3}$$

```
> #Problem 2
>
> LeslieMat([0.95, 0.97, 0.9], [0.1, 1.2, 0.9, 0.1])
```

$$\begin{bmatrix} 0.1 & 1.2 & 0.9 & 0.1 \\ 0.95 & 0 & 0 & 0 \\ 0 & 0.97 & 0 & 0 \\ 0 & 0 & 0.9 & 0 \end{bmatrix} \tag{4}$$

```
> Eigenvalues(Matrix([[0.1, 1.2, 0.9, 0.1], [0.95, 0, 0, 0], [0, 0.97, 0, 0], [0, 0, 0.9, 0]]))[1];
```

$$1.38573262885364 + 0.\,\mathrm{I} \tag{5}$$

```
> #The growth constants agree
>
> #Problem 3
> P := Matrix([[0.5, 0.5/3, 0.5/3, 0.5/3], [0.2, 0.4, 0.2, 0.2], [0.7/3, 0.7/3, 0.3, 0.7/3], [0.8/3, 0.8/3, 0.8/3, 0.2]])
```

$$P := \begin{bmatrix} 0.5 & 0.1666666667 & 0.1666666667 & 0.1666666667 \\ 0.2 & 0.4 & 0.2 & 0.2 \\ 0.2333333333 & 0.2333333333 & 0.3 & 0.2333333333 \\ 0.2666666667 & 0.2666666667 & 0.2666666667 & 0.2 \end{bmatrix} \tag{6}$$

```
>
> P^1000
```

(7)

$$\begin{bmatrix} 0.315197007189532 & 0.262664172672962 & 0.225140719443162 & 0.196998129500332 \\ 0.315197007144515 & 0.262664172635447 & 0.225140719411007 & 0.196998129472196 \\ 0.315197007112359 & 0.262664172608651 & 0.225140719388038 & 0.196998129452099 \\ 0.315197007176892 & 0.262664172662428 & 0.225140719434133 & 0.196998129492432 \end{bmatrix} \quad \textbf{(7)}$$

```
> #All rows are identical
> #0.315 stay at S1, 0.2627 stay at S2, 0.225 stay at S3, and 0.1970 stay at S4
> #Page Rank: S1, S2, S3, S4 in terms of popularity
>
```