OK TO POST HW4 Timothy Nuzzolillo

1. Give an example of a 2nd order linear equation w/ 2 specific solutions

$$y''(t) - 4y(t) = 0$$

Try $c_1 e^{2t} = y(t)$ $c_1(4e^{2t} - 4e^{2t}) = 0$ $y_1(t) = e^{2t}$

Try $c_2 e^{-2t} = y(t)$ $c_2(4e^{-2t} - 4e^{-2t}) = 0$ $y_2(t) = e^{-2t}$

$$y_3(t) = c_1 e^{2t} + c_2 e^{-2t}$$

$$= \underbrace{\cancel{4c_1 e^{2t}} + \cancel{4c_2 e^{-2t}}}_{y_3''(t)} \underbrace{- \cancel{4c_1 e^{2t}} - \cancel{4c_2 e^{-2t}}}_{-4y_3(t)} = 0$$

2. $y'(t)^2 - 4y(t) = 0$ Let $y_1(t) = t^2$

$(2t)^2 - 4t^2 = 0$

$4t^2 - 4t^2 = 0$ True!

$y_2(t) = 2t^2$ would not work since the differential equation is not linear, and a nonlinear change to a function w/ a constant makes you unable to factor it out

$y_2(t)$

$(y'(t))^2 - 4y(t) = 0$

$(4t)^2 - 4t^2 = 0$

$16t^2 - 4t^2 = 0$

$12t^2 = 0$ only works for $t = 0$

3 $a(n) = 5a(n-1) - 6a(n-2) \rightarrow a(n) - 5a(n-1) + 6(n-2) = 0$

Let $a(n) = b^n$ where b is any constant integer

$b^n - 5b^{n-1} + 6b^{n-2} = 0 \rightarrow b^{n-2}(b^2 - 5b + 6)$ $b = 2, 3$

$a_1(n) = 2^n \rightarrow 2^n - 5 \cdot 2^{n-1} + 6 \cdot 2^{n-2} = 0 \rightarrow 2^n - 5 \cdot 2^n \cdot 2^{-1} + 6 \cdot 2^n \cdot 2^{-2} = 0$

$2^n(1 - 2.5 + 1.5) = 0 \rightarrow 2(0) = 0$ ✓

$a_2(n) = 3^n \rightarrow 3^n - 5 \cdot 3^n \cdot \frac{1}{3} + 6 \cdot 3^n \cdot \frac{1}{9} \rightarrow 3^n(1 - \frac{5}{3} + \frac{6}{9}) = 3^n(0) = 0$ ✓

$a_3(n) = c_1 a_2(n) + c_2 a_1(n)$

$\rightarrow c_1 2^n + c_2 3^n - 5c_1 2^n \cdot \frac{1}{2} - 5c_2 3^n \frac{1}{3} + 6c_1 2^n \cdot \frac{1}{4} + 6c_2 3^n \cdot \frac{1}{9} = 0$

2^n Terms sum to 0, 3^n Terms sum to 0 Therefore $a_3(n) = 0$

4. $a(n) = a(n-1)^2$, $n \geq 0$

$a_1(n) = 2^{2^n}$ $\quad 2^{2^n} = (2^{2^{n-1}})^2 \rightarrow 2^{2^n} = 2^{2^{n-1} \cdot 2} \rightarrow 2^{2^n} = 2^{2^{n-1+1}} \rightarrow \underline{2^{2^n} - 2^{2^n} = 0}$

Works!

$a_2(n) = 3^{2^n}$ $\quad 3^{2^n} = (3^{2^{n-1}})^2 \rightarrow 3^{2^n} - 3^{2^{n-1} \cdot 2} = 0 \rightarrow 3^{2^n} - 3^{2^{n-1} \cdot 2} = 3^{2^n} - 3^{2^n} = 0$ Works!

$a_3(n)$ where $a_3(n) = a_1(n) + a_2(n)$, would not be another solution since $a(n)$ is nonlinear and the squaring of $a(n-1)$ would make a weird middle number that would not allow $a(n) - (a(n-1))^2$ to equal 0

$2^{2^n} + 3^{2^n} - (2^{2^{n-1}} + 3^{2^{n-1}})^2 \rightarrow 2^{2^n} + 3^{2^n} - (2^{2^{n-1+1}} + 2^{2^{n-1}} 3^{2^{n-1}} \cdot 2 + 3^{2^{n-1+1}})$

$2^{2^n} + 3^{2^n} - (2^{2^n} + 2^{2^n} 3^{2^{n-1}} + 3^{2^n}) = 0$ $\quad \boxed{2^{2^n} 3^{2^{n-1}} \neq 0}$

#5, Solve $a(n)-4a(n-2)=-3n+8$ $\quad a(0)=2$ $\quad a(1)=1$

Let $a(n)=r^n$

and apply to homog. $\quad r^n-4r^{n-2}=0=r^{n-2}(r^2-4)$ $\quad r=\pm 2$

Gen sol to homog $= c_1 2^n + c_2(-2)^n$

Particular soln, let $a(n)=\alpha n+\beta$

$$\alpha n+\beta-4\alpha n+8\alpha-4\beta=-3n+8$$

$$-3\alpha n=-3n \qquad 8\alpha-3\beta=8$$

$$\alpha=1 \qquad 8-3\beta=8$$

$$\beta=0$$

Gen soln $=c_1 2^n+c_2(-2)^n+n$

$a(0)=2=c_1 2^0+c_2(-2)^0$

$a(1)=1=2c_1-2c_2+1$

$$\begin{bmatrix} 2=c_1+c_2 \\ 0=2c_1-2c_2 \end{bmatrix} \rightarrow \begin{array}{r} 4=2c_1+2c_2 \\ +\ 0=2c_1-2c_2 \\ \hline 4=4c_1 \quad 0c_2 \end{array}$$

$c_1=1,\ c_2=1 \qquad c_1=1$

Final solution

$$\boxed{a(n)=2^n+(-2)^n+n}$$