> #OK to post homework
Shreya Ghosh, 09-20-2021, Assignment 4
#1.
#Second order differential equation: $y'' - 9y = 0$
#$r^2 - 9 = 0 \rightarrow r = 3, -3$
#$y1 = e^{3t}$, $y1' = 3e^{3t}$, $y1'' = 9e^{3t}$
#$y2 = e^{-3t}$, $y2' = -3e^{-3t}$, $y2'' = 9e^{-3t}$
#$(9e^{3t} + 9 3^{-3t}) - 9(e^{3t} + e^{-3t}) = 0 \rightarrow 9e^{3t} + 9 3^{-3t} - 9e^{3t} - 9e^{-3t} = 0 \rightarrow 0 = 0$

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> #2.
$y'(t)^2 - 4y(t) = 0$
#$y1(t) = t^2$, $y1'(t) = 2t$
#$(2t)^2 - 4(t^2) = 0 \rightarrow 4t^2 - 4t^2 = 0 \rightarrow 0 = 0$
#$y2(t) = 2y1(t) = 2t^2$ *is* **not** *a solution because the equation is nonlinear*
#$y2(t) = 2t^2$, $y2'(t) = 4t$
#$(4t)^2 - 4(2t^2) = 0 \rightarrow 16t^2 - 8t^2 = 0 \rightarrow 8t^2$ *does* **not** *equal* 0

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> #3.
#a(n)=a(n-1) +2a(n-2)
#$t^2 - t - 2 = 0 \rightarrow t = -1, 2$
#$a1 = -1^n$, $a2 = 2^n$
#$2^n + (-1)^n = (2^{n-1} + (-1)^{n-1}) + 2(2^{n-2} + (-1)^{n-2})$
#$2^n + (-1)^n = 2^n \cdot 2^{-1} + (-1)^n \cdot (-1)^{-1} + 2(2^n \cdot 2^{-2} + (-1)^n \cdot (-1)^{-2})$
#$2^n + (-1)^n = 2^n \cdot 2^{-1} - (-1)^n + 2(2^n \cdot 2^{-2} + (-1)^n)$
#$2^n + (-1)^n = 2^n \cdot 2^{-1} - (-1)^n + 2^n \cdot 2^{-1} + 2 \cdot (-1)^n$
#$2^n + (-1)^n = 2 \cdot (2^n \cdot 2^{-1}) + (-1)^n$
#$2^n + (-1)^n = 2^n + (-1)^n$

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> #4.
#$2^{2^n} = (2^{2^n - 1})^2 = 2^{2^n - 1 \cdot 2} = 2^{2^n}$
#$3^{2^n} = (3^{2^n - 1})^2 = 3^{2^n - 1 \cdot 2} = 3^{2^n}$
#The addition of solutions is not a solution because the recurrence is nonlinear
#$2^{2^n} + 3^{2^n} = (2^{2^n - 1} + 3^{2^n - 1})^2 = 2^{2^n} + 2(2^{2^n - 1})(3^{2^n - 1}) + 3^{2^n} \rightarrow$ **not** *equal* **to** *each other*

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> #5.
rsolve($\{f(n) = 7 \cdot f(n-1) - 12 \cdot f(n-2) + 6 \cdot n - 11, f(0) = 3, f(1) = 9\}, f(n)$);

$$3^n + 4^n + n + 1 \qquad \textbf{(1)}$$

> #a(n)-4a(n-2)=-3n +8
#homogenous solution: $C1 \cdot 2^n + C2 \cdot (-2)^n$

#(an+b)-4(a(n-2)+b)=-3n+8 -> -3an+(8a-3b)=-3n+8 -> a=1, b=0 -> particular solution: n

#a(n)=C1·2^n + C2·$(-2)^n$ + n

#2=C1+C2, 0=2·C1-2·C2 -> C1=1, C2=1

#a(n)=2^n + $(-2)^n$ + n

rsolve({$f(n)=4\cdot f(n-2)-3\cdot n+8, f(0)=2, f(1)=1$}, $f(n)$);

$$2^n + (-2)^n + n \qquad \textbf{(2)}$$

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