Max Mekhanikov HW 4 - ok to post

1) $y''(t) + y'(t) - 6y = 0$

$y = e^{rt}$, $y' = re^{rt}$, $y'' = r^2 e^{rt}$

$r^2 e^{rt} + re^{rt} - 6e^{rt} = 0$

$e^{rt}(r^2 + r - 6) = 0$

$r = 2, -3$

$y_1(t) = e^{2t}$, $y_2 = e^{-3t}$

$y(t) = C_1 e^{2t} + C_2 e^{-3t}$

$y'(t) = 2C_1 e^{2t} - 3C_2 e^{-3t}$, $y''(t) = 4C_1 e^{2t} + 9C_2 e^{-3t}$

$y''(t) + y'(t) - 6y = 0$

$\underline{4}C_1 e^{2t} + \underline{9}C_2 e^{-3t} + \underline{2}C_1 e^{2t} \underline{-3}C_2 e^{-3t} \underline{-6}(\underline{C_1 e^{2t}} + \underline{C_2 e^{-3t}}) = 0$

$0 = 0$ ✓

2) $y'(t)^2 - 4y(t) = 0$

$y' = \pm\sqrt{4y}$

$$y' = \pm 2\sqrt{y}$$

$$y' = 2\sqrt{y}.$$

$$\int dy/\sqrt{y} = \int 2\, dt$$

$$\int y^{-1/2}\, dy = 2t + C_1$$

$$2\sqrt{y} + C_2 = 2t + C_1$$

$$C_3 = C_1 - C_2$$

$$y_1 = t^2 + C_3 t + \frac{C_3^2}{4}$$

$$y' = -2\sqrt{y}.$$

$$\int dy/\sqrt{y} = -\int 2\, dt$$

$$\int y^{-1/2}\, dy = -(2t + C_1)$$

$$2\sqrt{y} + C_2 = -2t - C_1$$

$$y_2 = t^2 - C_3 t - \frac{C_3^2}{4}$$

By allowing $C_3 = 0$, we see $y(t) = t^2$ satisfies $y'(t)^2 - 4y(t) = 0$

$$y'(t)^2-4y(t)=0$$

$$(2\sqrt{y})^2-4(2t^2)=0$$

$$4(t^2)-8t^2=0$$

$$-4t^2 \neq 0$$

Although it may seem that 2y1(t)=2t^2 should be a solution at first glance based on linear combination, however the solutions have coefficients that combine to equal zero and only 2t^2 remains. After plugging this into the original equation above, we see this is not a correct solution.

3) $a(n)-5a(n-1)+6a(n-2)=0$

$$a(n)=R^n$$

$$R^n-5R^{n-1}+6R^{n-2}=0$$

$$R^{n-2}(R^2-5R+6)=0$$

$$(R-2)(R-3)=0 \rightarrow R=2,3$$

$$a(n)=C_1 2^n+C_2 3^n$$

$a(0)=1$
$a(1)=5$

$$1=C_1+C_2$$

$$5=2C_1+3C_2$$

$$5=2(1-C_2)+3C_2$$

$$5=2+C_2 \rightarrow C_2=3$$

$$C_1=-2$$

$$a(n)=-2(2^n)+3(3^n)$$

$a(n) - 5a(n-1) + 6a(n-2) = 0$

$\hookrightarrow a(n) = -2(2^n) + 3(3^n)$

$-2(2^n) + 3(3^n) - 5(-2(2^{n-1}) + 3(3^{n-1})) + 6a(n-2) = 0$

$-2(2^n) + 3(3^n) + 10(2^{n-1}) - 15(3^{n-1}) - 12(2^{n-2}) + 18(3^{n-2}) = 0$

$n = 0:$

$-2 + 3 + 5 - 5 - 12(1/4) + 2 = 0, \quad 0 = 0$ ✓

$n = 1:$

$-4 + 9 + 10 - 15 - 6 + 6 = 0, \quad 0 = 0$ ✓

True for all n.

4) $a(n) = a(n-1)^2, \; n \geq 0$

$a_1(n) = 2^{2^n}, \quad a_2(n) = 3^{2^n}$

$a_1(1) = 2^{2^1} = 4$

$a_1(2) = 2^{2^2} = 16 = a(1)^2$

$a_2(3) = 2^{2^3} = 256 = a(2)^2$

$a_2(1) = 3^{2^1} = 9$

$a_2(2) = 3^{2^2} = 81 = a_2(1)^2$

$a_3(3) = 3^{2^3} = 6561 = a_2(2)^2$

Both solutions satisfy $a(n) = a(n-1)^2, \; n \geq 0$

$a_3(n) = a_1(n) + a_2(n) = 2^{2^n} + 3^{2^n}$

$a_3(1) = 2^{2^1} + 3^{2^1} = 13$

$a_3(2) = 2^{2^2} + 3^{2^2} = 16 + 81 = 97 \neq 169 = a_3(1)^2$

a3(n) is not automatically a new solution because it does not satisfy the original recurrence equation as shown above.

5)

 # Max Mekhanikov - HW 5 - Ok to post

 # Question 5

rsolve$(\{a(n) - 7 \cdot a(n-1) + 12 \cdot a(n-2) = 6n - 11, a(0) = 3, a(1) = 9\}, a(n))$

$$3^n + n + 1 + 4^n$$

rsolve$(\{a(n) - 4 \cdot a(n-2) = -3 \cdot n + 8, a(0) = 2, a(1) = 1\}, a(n))$

$$2^n + (-2)^n + n$$

$a(n) - 4a(n-2) = -3n + 8$, $a(0) = 2$, $a(1) = 1$

$a(n) - 4a(n-2) = 0$

$y'' - 4y = 0 \rightarrow y = e^{rt}$

$(e^{rt})'' - 4e^{rt} = 0$

$r^2 e^{rt} - 4e^{rt} = 0 \rightarrow e^{rt}(r^2 - 4) = 0$

$r = \pm 2$

$y(t) = C_1 2^n + C_2(-2)^n$

$\alpha n + \beta - 4(\alpha(n-1) + \beta) = -3n + 8$

$\alpha n + \beta - 4\alpha n + 4 - 4\beta = -3n + 8$

$-3\alpha n - \beta = -3n + 4$

$\alpha = 1$, $\beta = 4$

$a(n) = C_1 2^n + C_2(-2)^n + n + 4$

$2 = C_1 + C_2 + n + 4$

$1 = 2C_1 - 2C_2 + n + 4$

$2 = C_1 + C_2 + n + 4$

$1 = 2C_1 - 2C_2 + n + 4$

$C_1 = -2 - C_2 - n$

$1 = 2(-2 - C_2 - n) - 2C_2 + n + 4$

$1 = \cancel{-4} - 2C_2 - 2n - 2C_2 + n + \cancel{4}$

$1 = -4C_2 - n$

$C_2 = -\frac{1+n}{4}$

$C_1 = -2 + \frac{1+n}{4} - n$

$a(n) = 2^n + (-2)^n + n$