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Anusha Nagar, 9.20.2021, Assignment 4

Anusha Nagar
Homework 4

1) $y'' - 5y' + 6y = 0$, $y(0) = 0$, $y'(0) = 1$

$(r^2 - 5r + 6) = 0$

$(r-3)(r-2) = 0 \Rightarrow r = 2, 3$

$y_1(t) = c_1 e^{2t}$, $y_2(t) = c_2 e^{3t}$

$$\left.\begin{array}{l} y(t) = c_1 e^{2t} + c_2 e^{3t} \Rightarrow c_1 + c_2 = 0 \\ y'(t) = 2c_1 e^{2t} + 3c_2 e^{3t} \Rightarrow 2c_1 + 3c_2 = 1 \end{array}\right\}$$

$c_1 = -1$, $c_2 = 1$

$y(t) = e^{3t} - e^{2t}$

$y'(t) = 3e^{3t} - 2e^{2t}$

$y''(t) = 9e^{3t} - 4e^{2t}$

$9e^{3t} - 4e^{2t} - 15e^{3t} + 10e^{2t} + 6e^{3t} - 6e^{2t} = 0$ ✓

$y(t) = e^{3t} - e^{2t}$ is a solution as well.

2) $y_1(t) = t^2 \Rightarrow y'(t)^2 - 4y(t) = 0$

$y_1'(t) = 2t \Rightarrow (2t)^2 - 4t^2 = 0$

$4t^2 - 4t^2 = 0$ ✓

$y_1(t)$ is a solution

$y_2(t) = 2\,y_1(t) = 2t^2$ is <u>NOT</u> a solution, as ~~the~~ it is not a linear differential equation.

$y_2'(t) = 4t \Rightarrow (4t)^2 - 4(2t^2) \overset{?}{=} 0$

$16t^2 - 8t^2 \neq 0$ ✓

$y_2(t)$ is not a solution

3) $a(n) - 3a(n-1) + 2a(n-2) = 0$

$\Rightarrow a_1(n) = 1$, $a_2(n) = 2$

$\hookrightarrow 1 - 3 + 2 = 0$ ✓ $\quad \hookrightarrow 2 - 6 + 4 = 0$ ✓

$a_3(n) = 3 \Rightarrow 3 - 9 + 6 = 0$ ✓

4) $a(n) = a(n-1)^2$, $n \geq 0$

$a_1(n) = 2^{2^n} \Rightarrow 2^{2^n} = \left(2^{2^{(n-1)}}\right)^2$

$a_3(n) = 2^{2^n} + 3^{2^n}$

$a_3(n-1) = 2^{2^{n-1}} + 3^{2^{n-1}}$

$2^{2^n} + 3^{2^n} = \left(2^{2^{n-1}} + 3^{2^{n-1}}\right)^2$

$= 2^{2^{(n-1)2}} + 2 \cdot$

5) Part i

$a(n) - 7a(n-1) + 12a(n-2) = 6n - 11$

$\Rightarrow a(0) = 3,\ a(1) = 9$

$\Rightarrow a(n) = R^n \Rightarrow R^n - 7R^{n-1} + 12R^{n-2} = 0$

$\Rightarrow R^{n-2}(R^2 - 7R + 12) = 0$

$(R-4)(R-3) \Rightarrow a(n) = c_1 \cdot 3^n + c_2 \cdot 4^n$

Part ii

Using Maple's rsolve command:

✓

Part iii

$a(n) - 4a(n-2) = -3n + 8$

$\Rightarrow a(0) = 2,\ a(1) = 1$

$a(n) = R^n \Rightarrow R^n - 4R^{n-2}$

$\hookrightarrow R^{n-2}(R^2 - 4) \Rightarrow R = -2, +2$

$\hookrightarrow a(n) = c_1 \cdot -2^n + c_2 \cdot 2^n$

$\alpha n + \beta - 4\underbrace{[\alpha(n-2) + \beta]}_{} = -3n + 8$

$\alpha n + \beta - 4[\alpha n - 2\alpha + \beta] = -3n + 8$

$\alpha n + \beta - 4\alpha n + 8\alpha - 4\beta = -3n + 8$

$-3\alpha n - 3\beta + 8\alpha = -3n + 8$

$\alpha = 1,\quad -3\beta + 8 = 8$

$\beta = 0$

$\hookrightarrow n$ is also a solution

$a(n) = c_1 \cdot -2^n + c_2 \cdot 2^n + n$

$\hookrightarrow a(0) = 2 = c_1 + c_2$

$\hookrightarrow a(1) = 1 = c_1 \cdot -2 + 2c_2 + 1$

$\hookrightarrow \begin{cases} 2 = c_1 + c_2 \\ 0 = 2c_2 - 2c_1 \Rightarrow c_1 = c_2 = 1 \end{cases}$

ANS: $(-2)^n + 2^n + n = a(n)$ ✓

Part IV

Solve using Maple

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> #Problem 5, Part II
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> $rsolve(\{a(n) - 7 * a(n-1) + 12 * a(n-2) = 6 \cdot n - 11, a(0) = 3, a(1) = 9\}, a(n))$

$$3^n + n + 1 + 4^n \tag{1}$$

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> #Problem 5, Part IV
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> $rsolve(\{a(n) - 4 * a(n-2) = -3 \cdot n + 8, a(0) = 2, a(1) = 1\}, a(n))$

$$(-2)^n + 2^n + n \tag{2}$$

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