OK to post
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1. $y'' - 7y' + 10y = 0$ has solutions

$y_1(t) = e^{5t}$ and $y_2(t) = e^{2t}$.

Let $y(t) = e^{5t} + e^{2t}$. Then,

$y'' - 7y' + 10y = 25e^{5t} + 4e^{2t} - 35e^{5t} - 14e^{2t}$

$+ 10e^{5t} + 10e^{2t} = 0$. Thus,

$y(t) = e^{5t} + e^{2t}$ is a solution.

2. $y_1(t) = t^2$

Then, $y_1'(t)^2 - 4y_1(t) = 4t^2 - 4t^2 = 0$, so

y_1 is a solution.

$y_2(t)$ is not necessarily a solution

because the differential equation is nonlinear.

$y_2'(t)^2 - 4y_2(t) = (4t)^2 - 4(2t^2) = 8t^2$

$\neq 0$, so y_2 is not a solution.

3. $a(n) - 7a(n-1) + 12a(n-2) = 0$

has solutions $a_1(n) = 4^n$ and $a_2(n) = 3^n$.

Let $a(n) = 4^n + 3^n$.

Then $a(n) - 7a(n-1) + 12a(n-2)$

$= 4^n + 3^n - 7 \cdot 4^{n-1} - 7 \cdot 3^{n-1} + 12 \cdot 4^{n-2}$

$+ 12 \cdot 3^{n-2} = 4^{n-2}(16 - 28 + 12) +$

$3^{n-2}(9 - 21 + 12) = 0.$

Thus, $a(n) = 4^n + 3^n$ is a solution.

4. $a(n) = a(n-1)^2, \quad n \geq 0$

$a_1(n) = 2^{2^n} = \left(2^{2^{n-1}}\right)^2$, so a_1 is a solution.

$a_2(n) = 3^{2^n} = \left(3^{2^{n-1}}\right)^2$, so a_2 is a solution.

$a_3(n)$ is not necessarily a solution because the recurrence relation is nonlinear.

$a_3(n) = 2^{2^n} + 3^{2^n}$

$$a_3(n-1)^2 = \left(2^{2^{n-1}} + 3^{2^{n-1}}\right)^2 = 2^{2^n} + 2\left(2^{2^{n-1}} + 3^{2^{n-1}}\right)$$

$$+ 3^{2^n} \neq a_3(n) \quad , \text{ so}$$

$a_3(n)$ is not a solution.

5. Part (i): $c_1 3^n + c_2 4^n$ is the general solution because 3 and 4 are the only solutions of the equation $r^2 - 7r + 12 = 0$, and any linear combination of solutions of a homogeneous linear recurrence is a solution.

Part (iii):

$$a(n) - 4a(n-2) = -3n + 8 \quad , \; a(0) = 2, \; a(1) = 1$$

$$a_h(n) - 4a_h(n-2) = 0$$

$$r^2 - 4 = 0$$

$$r = \pm 2$$

$$a_h(n) = c_1 2^n + c_2 (-2)^n$$

Try $\alpha n + \beta$

$$\alpha n + \beta - 4\alpha(n-2) + \beta = -3n + 8$$

$$-3\alpha n + 2\beta + 8\alpha = -3n + 8$$

$$\alpha = 1, \; \beta = 0$$

$$a(n) = c_1 2^n + c_2(-2)^n + n$$

$$a(0) = c_1 + c_2 = 2$$

$$a(1) = 2c_1 - 2c_2 + 1 = 1$$

$$c_1, c_2 = 1$$

$$\boxed{a(n) = 2^n + (-2)^n + n}$$