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*#Homework-3 maple part*
*#Dynamic Models in Biology - Dr.Z*

*#1. Use Maple to solve the following initial value problem differntial equations for 2 ≤ k ≤ 10*
*# Done at the end*

*#3. Doing it both by hand and via Maple, solve the following recurrence with the given initial*
*#conditions*

$rsolve(\{a(0)=2, a(1)=3, a(n)=3\cdot a(n-1)-2\cdot a(n-2)\}, a)$

$$1+2^n \tag{1}$$

*#4. Doing it both by hand and via Maple, solve the following recurrence with the given initial*
*#conditions*

$rsolve(\{a(0)=3, a(1)=2, a(2)=6, a(n)=2\,a(n-1)+2\,a(n-2)-2\,a(n-3)\}, a)$

$$\sum_{\_R=RootOf(2\_Z^3-2\_Z^2-2\_Z+1)} \left(-\frac{(-4\_R^2-4\_R+3)\left(\frac{1}{\_R}\right)^n}{(6\_R^2-4\_R-2)\_R}\right) \tag{2}$$

*#5. Doing it both by hand and via Maple solve the recurrence with the initial conditions*

$rsolve(\{a(0)=1, a(1)=0, a(2)=0, a(3)=0, a(n)=a(n-4)\}, a)$

$$\frac{1}{4}+\frac{(-\mathrm{I})^n}{4}+\frac{\mathrm{I}^n}{4}+\frac{(-1)^n}{4} \tag{3}$$

*#1: Use Maple to solve the following initial value problem differntial equations for 2 ≤ k ≤ 10*
*# Maple gets stuck when I try to execute k = 7*

$dsolve(\{D^{(2)}(y)(t)-y(t)=0, y(0)=1, D(y)(0)=0\}, y(t))$

$$y(t)=\frac{e^t}{2}+\frac{e^{-t}}{2} \tag{4}$$

$dsolve(\{D^{(3)}(y)(t)-y(t)=0, y(0)=1, D(y)(0)=0, D^{(2)}(y)(0)=0\}, y(t))$

$$y(t)=\frac{e^t}{3}+\frac{2\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} \tag{5}$$

$dsolve(\{D^{(4)}(y)(t)-y(t)=0, y(0)=1, D(y)(0)=0, D^{(2)}(y)(0)=0, D^{(3)}(y)(0)=0\}, y(t))$

$$y(t)=\frac{e^t}{4}+\frac{e^{-t}}{4}+\frac{\cos(t)}{2} \tag{6}$$

$dsolve(\{D^{(5)}(y)(t)-y(t)=0, y(0)=1, D(y)(0)=0, D^{(2)}(y)(0)=0, D^{(3)}(y)(0)=0, D^{(4)}(y)(0)=0\}, y(t))$

$$y(t)=\frac{256\,e^t}{(40+8\sqrt{5})(40-8\sqrt{5})} \tag{7}$$

$$+\frac{\left(16\sqrt{5}+16\right)\sqrt{5}\,\mathrm{e}^{\left(-\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\sqrt{5-\sqrt{5}}\,t}{4}\right)}{5\left(40+8\sqrt{5}\right)}$$

$$-\frac{\sqrt{5}\left(16-16\sqrt{5}\right)\mathrm{e}^{\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\sqrt{5+\sqrt{5}}\,t}{4}\right)}{5\left(40-8\sqrt{5}\right)}$$

$dsolve\left(\left\{\mathrm{D}^{(6)}(y)(t)-y(t)=0, y(0)=1, \mathrm{D}(y)(0)=0, \mathrm{D}^{(2)}(y)(0)=0, \mathrm{D}^{(3)}(y)(0)=0, \mathrm{D}^{(4)}(y)(0)=0, \mathrm{D}^{(5)}(y)(0)=0\right\}, y(t)\right)$

$$y(t)=\frac{\mathrm{e}^{t}}{6}+\frac{\mathrm{e}^{-t}}{6}+\frac{\mathrm{e}^{\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3}+\frac{\mathrm{e}^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} \tag{8}$$

*#was not able to find a single code that would iterate through k = 2..10*
*# Some of my trial code*

$seq\left(\mathrm{D}^{(k-1)}(y)(0)=0, k=2..6\right)$

$$\mathrm{D}(y)(0)=0, \mathrm{D}^{(2)}(y)(0)=0, \mathrm{D}^{(3)}(y)(0)=0, \mathrm{D}^{(4)}(y)(0)=0, \mathrm{D}^{(5)}(y)(0)=0 \tag{9}$$

$seq\left(dsolve\left(\left\{\mathrm{D}^{(k)}(y)(t)-y(t)=0, y(0)=1, seq\left(\left(\mathrm{D}^{(i)}(y)(0)=0\right), i=1..4\right)\right\}, y(t)\right), k=2..5\right)$

$$y(t)=\frac{256\,\mathrm{e}^{t}}{\left(40+8\sqrt{5}\right)\left(40-8\sqrt{5}\right)} \tag{10}$$

$$+\frac{\left(16\sqrt{5}+16\right)\sqrt{5}\,\mathrm{e}^{\left(-\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\sqrt{5-\sqrt{5}}\,t}{4}\right)}{5\left(40+8\sqrt{5}\right)}$$

$$-\frac{\sqrt{5}\left(16-16\sqrt{5}\right)\mathrm{e}^{\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\sqrt{5+\sqrt{5}}\,t}{4}\right)}{5\left(40-8\sqrt{5}\right)}$$

# Homework 3

1) Done in maple

2) $a(n) = a(n-1)^2$

$2^{2^n} = (2^{2^{(n-1)}})^2$

(non-linear recurrence)

3) $a(n) = 3a(n-1) + 2a(n-2)$ $\quad a(0)=2$

characteristic eq: $r^2 - 3r + 2 = 0$ $\quad a(1)=3$

$(r-1)(r-2)=0$

$r=1, 2$

$a_n = B2^n + D1^n$

$a_0 = 2 = B2^0 + D1^0$

$2 = B + D$

$a_1 = 3 = B2^1 + D1^1$

$3 = 2B + \cancel{D}$

$2 = B + \cancel{D}$

$B=1 \qquad D=1$

$a_n = 2^n + 1^n$ $\qquad$ for $(n \geq 0)$ $\boxed{a_n = 2^n + 1}$

4) Did using maple

5) $a(n) = a(n-4)$ $\qquad a(0)=1 \quad a(1)=0$

$= a(n-8)$ $\qquad a(2)=0 \quad a(3)=0$

$= a(n-16)$

$= a(n-20)$

$a(4) = a(n-4) = a_0 = 1$

$1, 0, 0, 0, 1, 0, 0, 0, 1 \ldots$

we get 1 after every 4 numbers

$\frac{1}{4} + \frac{(-i)^n}{4} + \frac{i^{n^2}}{4} + \frac{(-i)^{n^2}}{4}$

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