Dynamic Modeling HW3

3) $a(n) = 3a(n-1) - 2a(n-2)$; $a(0) = 2$, $a(1) = 3$

$1 = \frac{3}{r} - \frac{2}{r^2}$

$r^2\left(1 - \frac{3}{r} + \frac{2}{r^2} = 0\right)$

$r^2 - 3r + 2 = 0$

$(r-1)(r-2) = 0$

$a(n) = c_1 1^n + c_2 2^n$

$a(0) = (c_1 + c_2 = 2) - 1$

$a(1) = {}^{+} c_1 + 2c_2 = 3$

$c_2 = 1$

$c_1 + 1 = 2$

$c_1 = 1$

$a(n) = (1)^1(1)^n + (2^0)(2^n)$

$\boxed{a(n) = 1^{n+1} + 2^n}$

4) $a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3)$; $a(0) = 3$, $a(1) = 2$, $a(2) = 6$

$1 = \frac{2}{r} + \frac{2}{r^2} - \frac{2}{r^3}$

$\left(1 - \frac{2}{r} - \frac{2}{r^2} + \frac{2}{r^3} = 0\right) r^3$

$r^3 - 2r^2 - 2r + 2 = 0$

→ no roots for equation upon inspection

→ tried graphing equation

⇒ roots are: $(-1.17, 0)$, $(0.689, 0)$, $(2.481, 0)$

$a(n) = c_1(-1.17)^n + c_2(0.689)^n + c_3(2.481)^n$

$3 = c_1 + c_2 + c_3$

$2 = -1.17c_1 + 0.689c_2 + 2.481c_3$

$6 = 1.3689c_1 + 0.474721c_2 + 6.155361c_3$

→ used matrix math to solve (via calculator):

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ -1.17 & 0.689 & 2.481 & 2 \\ 1.369 & 0.475 & 6.155 & 6 \end{array}\right] \longrightarrow \left[\begin{array}{cccc} 1 & 0 & 0 & 0.238 \\ 0 & 1 & 0 & 1.994 \\ 0 & 0 & 1 & 0.768 \end{array}\right]$$

$\boxed{a(n) = 0.238(-1.17)^n + 1.994(0.689)^n + 0.768(2.481)^n}$

→ Maple was unable to compute, so I'm not sure if my methodology is correct

5) $a(n) = a(n-4)$; $a(0) = 1$, $a(1) = 0$, $a(2) = 0$, $a(3) = 0$

$1 = \frac{1}{r^4}$

$(1 - \frac{1}{r^4} = 0) r^4$

$r^4 - 1 = 0$

$(r^2+1)(r^2-1)$

$r = \pm i, \pm 1$

$a(n) = c_1 i^n + c_2(-i)^n + c_3(1)^n + c_4(-1)^n$

① $1 = c_1 + c_2 + c_3 + c_4$

② $0 = c_1 i - c_2 i + c_3 - c_4$

③ $0 = -c_1 + c_2 + c_3 + c_4$

④ $0 = -c_1 i + c_2 i + c_3 - c_4$

④ $0 = -c_1 i + c_2 i + c_3 - c_4$

$-c_2 i + c_1 i = c_3 - c_4$

$0 = c_1 i - c_2 i - c_2 i + c_1 i$

$0 = 2c_1 i - 2c_2 i$

$2c_1 i = 2c_2 i$

$c_1 = c_2$

② $0 = \cancel{c_1 i - c_1 i} + c_3 - c_4$

$\Rightarrow c_3 = c_4$

$\Rightarrow c_1 = c_2 = c_3 = c_4$

$1 = 4c_1 \Rightarrow c_1 = c_2 = c_3 = c_4 = \frac{1}{4}$

$$\boxed{a(n) = \frac{1}{4} i^n + \frac{1}{4}(-i)^n + \frac{1}{4}(1)^n + \frac{1}{4}(-1)^n}$$

2) * I wasn't quite sure how to prove this because we didn't talk much about non-linear recurrence equations in depth in class, but this is my best: *

(a) $2^{2^n} = a(n-1)^2$

$2^{2^n} = a(n^2-2n-1)$

$2^{2^n} = a(n^2) - a(2n) - a$?

(b) Constant multiples of the solution to this non-linear recurrence because this property only applies to linear homogeneous recurrence equations

> *#Do not post homework*
> *#Nikita John, September 13th, 2021, Assignment 3*

> *#3: Solve the following recurrence*
> $rsolve(\{a(n) - 3\,a(n-1) + 2\,a(n-2) = 0, a(0) = 2, a(1) = 3\}, \{a(n)\});$

$$\{a(n) = 1 + 2^n\} \tag{1}$$

> *#4: Solve the following recurrence*
> $rsolve(\{a(n) - 2\,a(n-1) - 2\,a(n-2) + 2\,a(n-3) = 0, a(0) = 3, a(1) = 2, a(2) = 6\}, \{a(n)\});$

$$\left\{a(n) = \sum_{\_R = RootOf(2\_Z^3 - 2\_Z^2 - 2\_Z + 1)} \left(-\frac{(-4\_R^2 - 4\_R + 3)\left(\frac{1}{\_R}\right)^n}{(6\_R^2 - 4\_R - 2)\,\_R}\right)\right\} \tag{2}$$

> *#5: Solve the following recurrence*
> $rsolve(\{a(n) - a(n-4) = 0, a(0) = 1, a(1) = 0, a(2) = 0, a(3) = 0\}, \{a(n)\});$

$$\left\{a(n) = \frac{1}{4} + \frac{I^n}{4} + \frac{(-I)^n}{4} + \frac{(-1)^n}{4}\right\} \tag{3}$$

> *#1: Solve the following non-linear differential equation*
> *# k = 2*
> $dsolve(\{D(D(y))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0\}, y(t));$

$$y(t) = \frac{e^{-t}}{2} + \frac{e^{t}}{2} \tag{4}$$

> $evalf(0.5 \cdot \exp(-1) + 0.5 \cdot \exp(1));$

$$1.543080635 \tag{5}$$

> *#k = 3*
> $dsolve(\{D(D(D((y))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0\}, y(t));$

$$y(t) = \frac{e^{t}}{3} + \frac{2\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} \tag{6}$$

> $evalf\left(\frac{1}{3} \cdot \exp(1) + \frac{2}{3} \cdot \left(\exp\left(-\frac{1}{2}\right) \cdot \cos\left(\frac{sqrt(3)(1)}{2}\right)\right)\right);$

$$1.168058313 \tag{7}$$

> *#k = 4*
> $dsolve(\{D(D(D(D(((y))))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0, D(D(D(y)))(0) = 0\}, y(t));$

$$y(t) = \frac{e^{-t}}{4} + \frac{e^{t}}{4} + \frac{\cos(t)}{2} \tag{8}$$

> $evalf(0.25 \cdot \exp(-1) + 0.25 \cdot \exp(1) + 0.5 \cdot \cos(1));$

$$1.041691470 \tag{9}$$

> *#k = 5*
> $dsolve(\{D(D(D(D(D((((y))))))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0, D(D(D(y)))(0) = 0, D(D(D(D(y))))(0) = 0\}, y(t));$

$$y(t) = \frac{256\,e^{t}}{(40 + 8\sqrt{5})(40 - 8\sqrt{5})} \tag{10}$$

$$+\frac{\left(16\sqrt{5}+16\right)\sqrt{5}\,\mathrm{e}^{\left(-\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\sqrt{5-\sqrt{5}}\,t}{4}\right)}{5\left(40+8\sqrt{5}\right)}$$
$$-\frac{\sqrt{5}\left(16-16\sqrt{5}\right)\mathrm{e}^{\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\sqrt{5+\sqrt{5}}\,t}{4}\right)}{5\left(40-8\sqrt{5}\right)}$$

> $evalf\left(\frac{256\,\mathrm{e}^{1}}{\left(40+8\sqrt{5}\right)\left(40-8\sqrt{5}\right)}+\frac{\left(16\sqrt{5}+16\right)\sqrt{5}\,\mathrm{e}^{\left(-\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\cdot(1)}\cos\left(\frac{\sqrt{2}\sqrt{5-\sqrt{5}}\cdot(1)}{4}\right)}{5\left(40+8\sqrt{5}\right)}-\frac{\sqrt{5}\left(16-16\sqrt{5}\right)\mathrm{e}^{\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)\cdot(1)}\cos\left(\frac{\sqrt{2}\sqrt{5+\sqrt{5}}\cdot(1)}{4}\right)}{5\left(40-8\sqrt{5}\right)}\right)$

$$1.008333609 \qquad \textbf{(11)}$$

> *#k = 6*
> *dsolve*({D(D(D(D(D(D(((((y))))))))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0, D(D(D(y)))(0) = 0, D(D(D(D(y))))(0) = 0, D(D(D(D(D(y)))))(0) = 0}, y(t));

$$y(t)=\frac{\mathrm{e}^{-t}}{6}+\frac{\mathrm{e}^{t}}{6}+\frac{\mathrm{e}^{\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3}+\frac{\mathrm{e}^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} \qquad \textbf{(12)}$$

> $evalf\left(\frac{\mathrm{e}^{(-1)}}{6}+\frac{\mathrm{e}^{(1)}}{6}+\frac{\mathrm{e}^{\frac{1}{2}}\cos\left(\frac{\sqrt{3}\cdot(1)}{2}\right)}{3}+\frac{\mathrm{e}^{-\frac{1}{2}}\cos\left(\frac{\sqrt{3}\cdot(1)}{2}\right)}{3}\right);$

$$1.001388891 \qquad \textbf{(13)}$$

> *#k = 7*

#At this point, hard coding the differential equations began to slow Maple down, however from the first 5 it can be seen that as k increases, y(1) decreases.

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