#Please do not post homework
#Julian Herman, 9/13/2021, Assignment 3

2) a.) $a_1(n) = 2^{2^n}$ to $a(n) = a(n-1)^2$

Base case: $n=0$, $a_1(0) = 2$

$n=1$, $a_1(1) = 2^2 = 4$

$n=2$, $a_1(2) = 2^{2^2} = 2^4 = 16$

$a(2) \stackrel{?}{=} a(2-1)^2$
$16 = 4^2$
$16 = 16$ ✓

$a(1) \stackrel{?}{=} a(1-1)^2$
$4 = 2^2$
$4 = 4$ ✓

Inductive step:

$$2^{2^n} = \left(2^{2^{(n-1)}}\right)^2$$

$$2^{2^n} = 2^{2^1 \cdot 2^{n-1}}$$

$$2^{2^n} = 2^{2^n}$$ ✓

b) $a_2(n) = 3 \cdot 2^{2^n} = 3 \cdot a_1(n)$

$a_2(0) = 3 \cdot 2 = 6$

$a_2(1) = 3 \cdot 4 = 12$

$a_3(2) = 3 \cdot 16 = 48$

$12 \neq 6^2$

$48 \neq 12^2$

$a_2(n)$, a constant multiple of $a_1(n)$, is **NOT** a solution to the non-linear recurrence because the recurrence equation is NON-LINEAR so the constant cannot just be factored out:

$$a_2(n) \stackrel{?}{=} a_2(n-1)^2$$

$$3 \cdot 2^{2^n} = \left(3 \cdot 2^{2^{n-1}}\right)^2$$

$$3 \cdot 2^{2^n} = 9 \cdot \left(2^{2^{n-1}}\right)^2$$

$$2^{2^n} = 3 \cdot \left(2^{2^{n-1}}\right)^2$$

$$a_1(n) \neq 3 \cdot a_1(n-1)^2$$

OR continued

$$2^{2^n} = 3 \cdot 2^{2^n}$$

$$1 \neq 3$$

3) $a(n) = 3a(n-1) - 2a(n-2), \quad a(0) = 2, \; a(1) = 3$

$$a(n) = r^n$$

$$r^n = 3 \cdot r^{(n-1)} - 2 \cdot r^{(n-2)}$$

$$r^n - 3 \cdot r^{(n-1)} + 2r^{(n-2)} = 0$$

$$r^{n-2}\left(r^2 - 3r + 2\right) = 0$$

$$(r-2)(r-1) = 0$$

$$r = 2, 1$$

$$a(n) = C_1 2^n + C_2 \cdot 1^n$$

$\leftarrow 1^n = 1$ for $n \in \mathbb{R}$

$a(0) = C_1 + C_2 = 2$ $\quad C_2 = 2 - C_1$

$a(1) = 2 \cdot C_1 + C_2 = 3$ $\quad \boxed{C_2 = 2 - 1 = 1}$

$2 \cdot C_1 + 2 - C_1 = 3$

$\boxed{C_1 = 1}$

$\boxed{\Rightarrow a(n) = 2^n + 1}$

5) $a(n) = a(n-4)$

$a(0) = 1$
$a(1) = 0$
$a(2) = 0$
$a(3) = 0$

$a(n) = r^n$

$r^n = r^{n-4}$

$r^n - r^{n-4} = 0$

$r^{n-4}(r^4 - 1) = 0$

$r^4 = 1$

$\boxed{r = 1, -1, i, -i}$

$a(n) = C_1 \cdot 1^n + C_2 \cdot (-1)^n + C_3 \cdot (i)^n + C_4 (-i)^n$

$a(0) = C_1 + C_2 + C_3 + C_4 = 1$

$a(1) = C_1 - C_2 + C_3 i - C_4 \cdot i = 0$

$a(2) = C_1 + C_2 - C_3 - C_4 = 0$

$a(3) = C_1 - C_2 + C_3 \cdot i^3 - C_4 \cdot i^3 = 0$

$$= c_1 - c_2 - c_3 \cdot i + c_4 i = 0$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & i & -i \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -i & i \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 2 & -2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 & 1 \\ 1 & -1 & -i & i & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1/2 \\ 0 & 2 & 1+i & 1-i & 1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 1 & 1 \\ 0 & 1 & \frac{1+i}{2} & \frac{1-i}{2} & 1/2 \\ 0 & 0 & 1 & 1 & 1/2 \end{array}\right]$$

$$\begin{array}{c} A) \\ B) \\ C) \\ D) \end{array} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1/2 & 1/2 & 1/2 \\ 0 & 0 & i/2 & -i/2 & 0 \\ 0 & 0 & 1 & 1 & 1/2 \end{array}\right]$$

D) $C_3 + C_4 = \frac{1}{2}$

c) $\frac{i}{2} \cdot C_3 - \frac{i}{2} C_4 = 0 \Rightarrow \cancel{\frac{i}{2}} C_3 = \cancel{\frac{i}{2}} C_4$

$C_3 = C_4$

$C_3 + C_3 = \frac{1}{2}$

$2C_3 = \frac{1}{2}$

$\boxed{C_3 = \frac{1}{4}, \quad C_4 = \frac{1}{4}}$

B) $C_2 + \frac{1}{2} C_3 + \frac{1}{2} C_4 = \frac{1}{2}$

$C_2 + \frac{1}{4} = \frac{1}{2}$

$\boxed{C_2 = \frac{1}{4}}$

A) $C_1 + C_2 + C_3 + C_4 = 1$

$C_1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$

$\boxed{C_1 = \frac{1}{4}}$

$$\boxed{\Rightarrow a(n) = \frac{1}{4} + \frac{(-1)^n}{4} + \frac{i^n}{4} + \frac{(-i)^n}{4}}$$

4) $\Rightarrow$ Maple couldn't solve it... no solutions?

$$r^n = 2r^{n-1} + 2r^{n-2} - 2r^{n-3}$$

$$2r^{n-3}\left(\frac{1}{2}r^3 - r^2 - r + 1\right)$$

$$\frac{1}{2}r^3 - r^2 - r + 1 = 0$$

...?