

HW 3

Ok topast Homework

Jeton Hida, 9/13/21, Assignment 3

3. $a_n = 3a_{n-1} - 2a_{n-2}$ $a(0) = 2, a(1) = 3$

$$a_n - 3a_{n-1} + 2a_{n-2} = 0$$

$$r^n - 3r^{n-1} + 2r^{n-2} = 0$$

$$r^n \cdot (1 - 3r^{-1} + 2r^{-2}) = 0$$

$$1 - \frac{3}{r} + \frac{2}{r^2} = 0$$

$$r^2 - 3r + 2 = 0$$

$$(r-2)(r-1) = 0$$

$$r=2, r=1$$

$$a_n = c_1 \cdot 2^n + c_2 \cdot 1^n$$

$$a_0 = c_1 + c_2 = 2$$

$$a_1 = 2c_1 + c_2 = 3$$

$$-1c_1 = -1$$

$$c_1 = 1, c_2 = 1$$

$$\varepsilon = a_n = 2^n + 1^n$$

4. $a_n = 2a_{n-1} + 2a_{n-2} - 2a_{n-3}$ $a(0) = 3, a(1) = 2, a(2) = 6$

$$a_n - 2a_{n-1} - 2a_{n-2} + 2a_{n-3} = 0$$

$$r^n - 2r^{n-1} - 2r^{n-2} + 2r^{n-3} = 0$$

$$r^n \cdot (1 - 2r^{-1} - 2r^{-2} + 2r^{-3}) = 0$$

$$1 - \frac{2}{r} - \frac{2}{r^2} + \frac{2}{r^3} = 0$$

$$r^3 - 2r^2 - 2r + 2 = 0$$

$$r^3 - 2r^2 - 2r + 2 = 0$$

$$r^3 + \varepsilon = 1$$

$$r^3 - \varepsilon = 1$$

$$\begin{bmatrix} 1 & -\varepsilon \\ \varepsilon & 1 \end{bmatrix}$$

$$5. \quad a_n = a_{n-4} \quad a(0) = 1, a(1) = 0, a(2) = 0, a(3) = 0$$

$$a_n - a_{n-4} = 0$$

$$r^n - r^{n-4} = 0$$

$$r^n \cdot (1 - r^{-4}) = 0$$

$$r^4 - 1 = 0$$

$$(r^2 + 1)(r^2 - 1)$$

$$(r^2 + 1)(r + 1)(r - 1)$$

$$r = \pm i, r = -1, r = 1$$

$$a_n = C_1 \cdot i^n + C_2 \cdot -i^n + C_3 \cdot -1^n + C_4 \cdot 1^n$$

$$a(0) = C_1 + C_2 + C_3 + C_4 = 1$$

$$a(1) = C_1 i + C_2 -i - C_3 + C_4 = 0$$

$$a(2) = -C_1 + C_2 + C_3 + C_4 = 0$$

$$a(3) = -C_1 i + C_2 i - C_3 + C_4 = 0$$

$$a_n = \frac{i^n}{4} + \frac{-i^n}{4} + \frac{1^n}{4} + \frac{-1^n}{4}$$