HW 3

3.

$$a(n) = 3a(n-1) - 2a(n-2)$$

$$a(n) - 3a(n-1) + 2a(n-2) = 0$$

$$a(n) = r^n$$

$$r^n - 3r^{n-1} + 2r^{n-2} = 0$$

$$r^2 - 3r + 2 = 0 \qquad \text{Divide by } r^{n-2}$$

$$(r-1)(r-2) = 0$$

$$r = 1, 2$$

$$a(n) = c1 + c2 \cdot 2^n$$

② $a(0) = c1 + c2 = 2$

① $a(1) = c1 + 2 \cdot c2 = 3$

① − ② $\quad c2 = 1$

$c1 = 1$

$$\boxed{a(n) = 1 + 2^n}$$

4.

$a(n) - 2a(n-1) + 2a(n-2) - 2a(n-3) = 0$

$a(n) = r^n$

$r^n - 2r^{n-1} + 2r^{n-2} - 2r^{n-3} = 0$

$r^3 - 2r^2 + 2r - 2 = 0$ Divide by r^{n-3}

$r_1 = 1.54369$ $r_{2,3} = 0.22816 \pm 1.11514\, i$

① $c1 \cdot (1.54369)^0 + c2 \cdot 1 + c3 \cdot 1 = 3$

② $c1 \cdot (1.54369) + c2 \cdot (.228 + 1.15i) + c3 \cdot (.228 - 1.15i) = 2$

③ $c1 \cdot (1.54369)^2 + c2 \cdot (.228 + 1.15i)^2 + c3 \cdot (.228 - 1.15i)^2 = 6$

c2 and c3 must be positive and equal for ② to be a real number and ③ to be a real number

$c2 = c3$

Guess: $c1 = c2 = c3 = 1$

For ② $1.54369 + .228 + .228 = 2$ ✓

For ① $1 + 1 + 1 = 3$ ✓

For ③ $1.54369^2 + .228^2 + \cancel{(1.15^2)(-1)} + .228^2 + \cancel{(1.15)^2(-1)}$

Maybe $a(n) = 1.54^n + (.228 + 1.15i)^n + (.228 - 1.15i)^n$

5. $a(n) - a(n-4) = 0$

$r^n - r^{n-4} = 0$

$r^4 - 1 = 0$

$(r^2-1)(r^2+1) = 0$

$(r+1)(r-1)(r^2+1) = 0$

$r = -1, 1, \pm i$

$c_1(-1)^n + c_2 + c_3 \cdot i^n + c_4(-i)^n = a(n)$

① $c_1 + c_2 + c_3 + c_4 = 1$

② $-c_1 + c_2 + \cancel{c_3 \cdot i} + \cancel{c_4 \cdot (-i)} = 0$

③ $c_1 + c_2 + (-c_3) - c_4 = 0$

④ $-c_1 + c_2 - \cancel{c_3 \cdot i} + \cancel{c_4 \cdot i} = 0$

$c_1 = c_2 = c_3 = c_4 = \frac{1}{4}$ → Checks out

$$\boxed{a(n) = \frac{(-1)^n}{4} + \frac{1}{4} + \frac{i^n}{4} + \frac{(-1)^n}{4}}$$