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3. Solve recurrence by hand

$a(n) = 3a(n-1) - 2a(n-2)$, $a(0)=2$, $a(1)=3$

Let $a(n) = 1$

$a(n-1) = \frac{1}{r}$

$a(n-2) = \frac{1}{r^2}$

thus, $1 = \frac{3}{r} - \frac{2}{r^2}$

$\Rightarrow \frac{1}{1} - \frac{3}{r} + \frac{2}{r^2} = 0$

multiply everything by r^2 to get

$r^2 - 3r + 2 = 0$

$\Rightarrow (r-2)(r-1) = 0$

Now we have $a_n = x2^n + y1^n$

And we will use our initial conditions to find our values of x and y.

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3 (continued)

Given $a_n = x2^n + y1^n$,

$$\begin{cases} a(0) \equiv 2 = x + y \\ a(1) = 3 = 2x + y \end{cases}$$

Thus, $x = 1$ and $y = 1$

Thus,

$$a(n) = 2^n + 1 .$$

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4. $a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3)$

$a(0) = 3 \quad a(1) = 2 \quad a(2) = 6$

let $a(n) = 1$

$a(n-1) = \frac{1}{r}$

$a(n-2) = \frac{1}{r^2}$

$a(n-3) = \frac{1}{r^3}$

Thus,

$r^3 - 2r^2 - 2r + 2 = 0$

$r^2(r-2) - 2(r-2) = 0$

$(r^2-2)(r-2) = 0$

thus, $r = \pm\sqrt{2}$ and $r = 2$

Thus,

we have our equation

$a(n) = C_1\sqrt{2}^n - C_2\sqrt{2}^n + C_3 2^n$

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4. Then, we have $c_1\sqrt{2}^n - c_2\sqrt{2}^n + c_3 2^n = 2a(n-1) + 2a(n-2) - 2a(n-3)$

Then, we still can have Difference quotients with discrete values

Or have ~~1st equation~~ save for later

$$c_1\sqrt{2}^n - c_2\sqrt{2}^n + c_3 2^n = \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$$

Which is

$$r^3 c_1\sqrt{2}^n + r^3 c_2\sqrt{2}^n + r^3 c_3 2^n = 2r^3 + 2r^2 - 2r$$

Which is $r^3(c_1\sqrt{2}^n - c_2\sqrt{2}^n + c_3 2^n - 2) = 2r(r-1)$

$n=0$:

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4. $c_1\sqrt{2}^n - c_2\sqrt{2}^n + c_3 2^n$

Let $c_1 = c_2$ to make things easy. and c_3 = something else.

Now we only have 2 unknown coefficients!

Because n also has odds,