```
> #OK to post
> #Anusha Nagar, 9.13.2021, Assignment 3
>
> #Problem 1
> #For k=2:
> dsolve({D(D(y))(t)-y(t)=0, y(0)=1, D(y)(0)=0}, y(t))
```

$$y(t) = \frac{e^{t}}{2} + \frac{e^{-t}}{2} \tag{1}$$

```
> #For k=3:
> dsolve({D(D(D(y)))(t)-y(t)=0, y(0)=1, D(y)(0)=0, D(D(y))(0)=0},
      y(t))
```

$$y(t) = \frac{e^{t}}{3} + \frac{2\,e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} \tag{2}$$

```
> #For k=4:
> dsolve({D(D(D(D(y))))(t)-y(t)=0, y(0)=1, D(y)(0)=0, D(D(y))(0)=0,
      D(D(D(y)))(0)=0}, y(t))
```

$$y(t) = \frac{e^{t}}{4} + \frac{e^{-t}}{4} + \frac{\cos(t)}{2} \tag{3}$$

```
> #For k=5:
> dsolve({D(D(D(D(D(y)))))(t)-y(t)=0, y(0)=1, D(y)(0)=0, D(D(y))(0)
      =0, D(D(D(y)))(0)=0, D(D(D(D(y))))(0)=0}, y(t))
```

$$y(t) = \frac{256\,e^{t}}{\left(40+8\sqrt{5}\right)\left(40-8\sqrt{5}\right)} + \frac{\left(16\sqrt{5}+16\right)\sqrt{5}\,e^{\left(-\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\,\sqrt{5-\sqrt{5}}\,t}{4}\right)}{5\left(40+8\sqrt{5}\right)} - \frac{\sqrt{5}\left(16-16\sqrt{5}\right)e^{\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)t}\cos\left(\frac{\sqrt{2}\,\sqrt{5+\sqrt{5}}\,t}{4}\right)}{5\left(40-8\sqrt{5}\right)} \tag{4}$$

```
> #For k=6:
> dsolve({D(D(D(D(D(D(y)))))))(t)-y(t)=0, y(0)=1, D(y)(0)=0, D(D(y))(0)
      =0, D(D(D(y)))(0)=0, D(D(D(D(y))))(0)=0, D(D(D(D(D(y)))))(0)=0},
      y(t))
```

$$y(t) = \frac{e^{t}}{6} + \frac{e^{-t}}{6} + \frac{e^{\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} + \frac{e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} \tag{5}$$

```
> #For k=7:
> dsolve({D(D(D(D(D(D(D(y)))))))(t)-y(t)=0, y(0)=1, D(y)(0)=0,
      D(D(y))(0)=0, D(D(D(y)))(0)=0, D(D(D(D(y))))(0)=0,
      D(D(D(D(D(y)))))(0)=0, D(D(D(D(D(D(y))))))(0)=0}, y(t))
```

```
> #For above, when we got to k=7, Maple just kept running and got
    stuck trying to compute.
>
> #Problem 2
> #Problem (a)

> #We check whether 2^(2^n) = 2^(2^(n-1)^2)
> expand(2^(2^(n-1)^2));
```

$$\left(\frac{2^{n^2}}{2^{(2^n)^2}}\right)^2 \tag{6}$$

```
> #a2(n) is not a solution. We have a non-linear recurrence here
    where we are squaring the term (n-1) in the recurrance
    equation. Just as with diffrential equations, we cannot take
    multiples of a solution as a solution when the terms are being
    squared.
> #Problem 3:
> #By HAND: a(2) = 3·3-2·2 = 5. a(3) = 3·5-2·3 = 9. a(4) = 3·9-2·5 =
    17. Here, we note that the differences between numbers are 1,
    2, 4, 8, which just by inspection leads me to believe that the
    explicit formula is 2^(n)+1.
> #By Maple:
> a:=proc(n) option remember: if n=0 then 2 elif n=1 then 3:
    else 3·a(n-1)-2·a(n-2): fi: end:
> seq(a(i), i=0..10);
```

$$2, 3, 5, 9, 17, 33, 65, 129, 257, 513, 1025 \tag{7}$$

```
>
>
> #Problem 4
> #By hand: a(3) = 2·6+2·2-2·3 = 10. a(4) = 2·10+2·6-2·2 = 28. a(5)
    = 2·28+2·10-2·6 = 64. I don't know how to finish this problem,
    I don't see any relationships between the numbers.
> #By Maple:
> a:=proc(n) option remember: if n=0 then 3 elif n=1 then 2 elif n
    =2 then 6: else 2·a(n-1)+2·a(n-2)-2·a(n-3): fi: end:
> seq(a(f), f=0..10);
```

$$3, 2, 6, 10, 28, 64, 164, 400, 1000, 2472, 6144 \tag{8}$$

```
>
>
> #Problem 5
> #By hand: a(4) = 1, a(5) = 0, a(6) = 0, a(7) = 0, a(8) = 1. Here,
    if n is divisible by 4, then a(n) = 1; else, a(n) = 0.
> #By maple:
> a:=proc(n) option remember: if n=0 then 1 elif n = 1 then 0
    elif n = 2 then 0 elif n = 3 then 0: else a(n-4): fi: end:
> seq(a(i), i=0..10);
```

$$1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0 \tag{9}$$