> #OK to post

> #Anne Somalwar, 9.13.2021, hw3

>

> #1

> #Use Maple to solve the following initial value problem differntial equations for $2 \le k \le 10$
> # $y^{(k)}(t) - y(t) = 0 \quad ; \quad y(0) = 1, \quad y'(0) = 0, \quad \ldots, \quad y^{(k-1)}(0) = 0$.
> #and then find the value (**in** decimals) of $y(1)$. Do you see a trend?

>

> #Create list of k-1 initial conditions

> $G(k) := y(0) = 1, seq\left(D^{(i)}(y)(0) = 0, i = 1..(k-1)\right)$

$$G := k \mapsto \left(y(0) = 1, seq\left(D^{(i)}(y)(0) = 0, i = 1..k-1\right)\right) \quad \textbf{(1)}$$

> #k=2

> $dsolve\left(\{ D^{(2)}(y)(t) - y(t) = 0, G(2)\}, y(t)\right)$

$$y(t) = \frac{e^{-t}}{2} + \frac{e^{t}}{2} \quad \textbf{(2)}$$

> #I renamed y(t) s(t) to avoid confusing maple

> $s(t) := \frac{e^{-t}}{2} + \frac{e^{t}}{2}$

$$s := t \mapsto \frac{e^{-t}}{2} + \frac{e^{t}}{2} \quad \textbf{(3)}$$

> $evalf(s(1))$

$$1.543080635 \quad \textbf{(4)}$$

> #k=3

> $dsolve\left(\{ D^{(3)}(y)(t) - y(t) = 0, \ y(0) = 1, G(3)\}, y(t)\right)$

$$y(t) = \frac{e^{t}}{3} + \frac{2\, e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right)}{3} \quad \textbf{(5)}$$

> $s(t) := \frac{e^{t}}{3} + \frac{2\, e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right)}{3}$

$$s := t \mapsto \frac{e^{t}}{3} + \frac{2 \cdot e^{-\frac{t}{2}} \cdot \cos\left(\frac{\sqrt{3} \cdot t}{2}\right)}{3} \quad \textbf{(6)}$$

> $evalf(s(1))$

$$1.168058313 \quad \textbf{(7)}$$

> #k=4

> $dsolve\left(\{ D^{(4)}(y)(t) - y(t) = 0, \ y(0) = 1, G(4)\}, y(t)\right)$

$$y(t) = \frac{e^{-t}}{4} + \frac{e^{t}}{4} + \frac{\cos(t)}{2} \quad \textbf{(8)}$$

> $s(t) := \frac{e^{-t}}{4} + \frac{e^{t}}{4} + \frac{\cos(t)}{2}$

$$s := t \mapsto \frac{e^{-t}}{4} + \frac{e^{t}}{4} + \frac{\cos(t)}{2} \quad (9)$$

> *evalf*$(s(1))$

$$1.041691470 \quad (10)$$

> *#For k = 5,..., 10, I found y(t) the same way but the results were too long to include.*

> *# for k= 5, y(1) = 1.008333609*

> *#for k=6, y(1) = 1.001388891*

> *#for k=7, it would not run for some reason.*

> *#for k=8, y(1) = 1.000024802*

> *#for k= 9, y(1) = 1.000002756*

> *#for k=10, y(1) = 1.000000275*

> *#Trend: y(1) gets closer and closer to 1.*

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> *#3*

> *#Doing it both by hand and via Maple, solve the following recurrence with the given initial conditions*
> # $a(n) = 3\,a(n-1) - 2\,a(n-2)$; $a(0) = 2$, $a(1) = 3$

>

> *rsolve*$(\{a(n) = 3 \cdot a(n-1) - 2 \cdot a(n-2), a(0) = 2, a(1) = 3\}, a(n));$

$$1 + 2^{n} \quad (11)$$

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> *#4*

> *#Doing it both by hand and via Maple, solve the following recurrence with the given initial conditions*
> #$a(n) = 2\,a(n-1) + 2\,a(n-2) - 2\,a(n-3)$; $a(0) = 3$, $a(1) = 2$, $a(2) = 6$

>

> $a(n) = a(n)$

$$a(n) = a(n) \quad (12)$$

> *rsolve*$(\{a(n) = 2 \cdot a(n-1) + 2 \cdot a(n-2) - 2 \cdot a(n-3), a(0) = 3, a(1) = 2, a(2) = 6\},$
> $a(n));$

Error, (in genfunc:-rgf_expand) unable to compute coeff

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> #5

> #Doing it both by hand and via Maple solve the recurrence with the initial conditions
$a(n) = a(n-4)$; $a(0) = 1, a(1) = 0, a(2) = 0, a(3) = 0$.

>

> $a(n) = a(n)$

$$a(n) = a(n) \tag{13}$$

> rsolve({ a(n) = a(n - 4), a(0) = 1, a(1) = 0, a(2) = 0, a(3) = 0}, a(n));

$$\frac{I^n}{4} + \frac{(-I)^n}{4} + \frac{(-1)^n}{4} + \frac{1}{4} \tag{14}$$

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