OK to post

Anne Somalwar, 9/13/2021, Assignment 3

#2 $a(n) = a(n-1)^2$

$$2^{2^n} = \left(2^{2^{n-1}}\right)^2 = 2^{2^n},$$

So a_1 is a solution.

a_2 is not a solution because

$$3 \cdot 2^{2^n} \neq \left(3 \cdot 2^{2^{n-1}}\right)^2 = 9 \cdot 2^{2^n}.$$

#3

$$a(n) = 3a(n-1) - 2a(n-2)$$

$$a(n) = r^n$$

$$r^n = 3r^{n-1} - 2r^{n-2}$$

$$r^2 = 3r - 2$$

$$r^2 - 3r + 2 = 0$$

$$r = 2, 1$$

$$\boxed{a(n) = 1 + 2^n}$$

#4

$$a(n) = 2a(n-1) + 2a(n-2) - 2a(n-3)$$

$$r^n = 2r^{n-1} + 2r^{n-2} - 2r^{n-3}$$

$$r^3 = 2r^2 + 2r - 2$$

$$r^3 - 2r^2 - 2r + 2 = 0$$

Using the computer,

$r \approx -1.1701,\ 0.68889,\ 2.4812$

$$a(n) = (-1.1701)^n + (0.68889)^n + (2.4812)^n$$

This does not satisfy $a(2) = 6$, but I'm not sure how to alter it.

#5

$$a(n) = a(n-4)$$

$$r^n = r^{n-4}$$

$$r^4 = 1$$

$$r = 1, -1, i, -i$$

$$a(n) = 1 + (-1)^n + i^n + (-i)^n$$

$a(0) = 1$, $a(1) = 0$, $a(2) = 0$,

$a(3) = 0$

$$\boxed{a(n) = \frac{1 + (-1)^n + i^n + (-i)^n}{4}}$$