\> # Max Mekhanikov - Homework 3 - Okay to share

\>

\> # ***Question*** 1

\>

$\# y^k(t) - y(t) = 0,\ \ y(0) = 1,\ \ y'(0) = 0,\ \ y^{k-1}(0) = 0$

k = 2

\> $dsolve(\{D(D(y))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0\}, y(t));$

$$y(t) = \frac{e^t}{2} + \frac{e^{-t}}{2} \quad (1)$$

\> $evalf\left(y(1) = \frac{e^1}{2} + \frac{e^{-1}}{2}\right)$

$$y(1) = 1.543080635 \quad (2)$$

\> # ***k=3***

\> $dsolve(\{D(D(D(y)))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0\}, y(t));$

$$y(t) = \frac{e^t}{3} + \frac{2\, e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}\, t}{2}\right)}{3} \quad (3)$$

\> $evalf\left(y(1) = \frac{e^1}{3} + \frac{2\, e^{-\frac{1}{2}} \cos\left(\frac{\sqrt{3}}{2}\right)}{3}\right)$

$$y(1) = 1.168058313 \quad (4)$$

\> # ***k = 4***

\> $dsolve(\{D(D(D(D(y))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0, D(D(D(y)))(0) = 0\}, y(t));$

$$y(t) = \frac{e^t}{4} + \frac{e^{-t}}{4} + \frac{\cos(t)}{2} \quad (5)$$

\> $evalf\left(y(1) = \frac{e^1}{4} + \frac{e^{-1}}{4} + \frac{\cos(1)}{2}\right)$

$$y(1) = 1.041691470 \quad (6)$$

\> # ***k = 5***

\> $dsolve(\{D(D(D(D(D(y)))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0, D(D(D(y)))(0) = 0, D(D(D(D(y))))(0) = 0\}, y(t));$

$$y(t) = \frac{256\, e^t}{(40 + 8\sqrt{5})(40 - 8\sqrt{5})} + \frac{(16\sqrt{5} + 16)\sqrt{5}\, e^{\left(-\frac{\sqrt{5}}{4} - \frac{1}{4}\right) t} \cos\left(\frac{\sqrt{2}\sqrt{5 - \sqrt{5}}\, t}{4}\right)}{5\,(40 + 8\sqrt{5})} - \frac{\sqrt{5}\,(16 - 16\sqrt{5})\, e^{\left(\frac{\sqrt{5}}{4} - \frac{1}{4}\right) t} \cos\left(\frac{\sqrt{2}\sqrt{5 + \sqrt{5}}\, t}{4}\right)}{5\,(40 - 8\sqrt{5})} \quad (7)$$

> $evalf\left(y(1) = \frac{256\,e}{(40+8\sqrt{5})(40-8\sqrt{5})} + \frac{(16\sqrt{5}+16)\sqrt{5}\,e^{\left(-\frac{\sqrt{5}}{4}-\frac{1}{4}\right)}\cos\left(\frac{\sqrt{2}\sqrt{5-\sqrt{5}}}{4}\right)}{5(40+8\sqrt{5})} - \frac{\sqrt{5}(16-16\sqrt{5})\,e^{\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)}\cos\left(\frac{\sqrt{2}\sqrt{5+\sqrt{5}}}{4}\right)}{5(40-8\sqrt{5})} \right);$

$$y(1) = 1.008333609 \quad (8)$$

> # ***k* = 6**

> $dsolve(\{D(D(D(D(D(D(y))))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0, D(D(D(y)))(0) = 0, D(D(D(D(y))))(0) = 0, D(D(D(D(D(y)))))(0) = 0\}, y(t));$

$$y(t) = \frac{e^{t}}{6} + \frac{e^{-t}}{6} + \frac{e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} + \frac{e^{\frac{t}{2}}\cos\left(\frac{\sqrt{3}\,t}{2}\right)}{3} \quad (9)$$

> $evalf\left(y(1) = \frac{e}{6} + \frac{e^{-1}}{6} + \frac{e^{-\frac{1}{2}}\cos\left(\frac{\sqrt{3}}{2}\right)}{3} + \frac{e^{\frac{1}{2}}\cos\left(\frac{\sqrt{3}}{2}\right)}{3} \right);$

$$y(1) = 1.001388891 \quad (10)$$

> # ***k* = 7**
> $dsolve(\{D(D(D(D(D(D(D(y)))))))(t) - y(t) = 0, y(0) = 1, D(y)(0) = 0, D(D(y))(0) = 0, D(D(D(y)))(0) = 0, D(D(D(D(y))))(0) = 0, D(D(D(D(D(y)))))(0) = 0, D(D(D(D(D(D(y))))))(0) = 0\}, y(t));$

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At k=7, the computations become too complex and Maple gets stuck on "evaluating..." indefinitely. We see the trend of y(1) decimal values decrease with each increasing k iteration however.

> # ***Question 2***

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> $rsolve(\{a(n) = a(n-1)^2, a(0) = 2\}, a(n));$

$$2^{2^n} \quad (11)$$

> # Here we see that if we set the initial value of a(0)=2, the result proves a(n)=2^2^n satisfies the non-linear recurrance equation.

> $rsolve(\{a(n) = a(n-1)^2, a(0) = 6\}, a(n));$

$$2^{2^n}\,3^{2^n} \quad (12)$$

> # By setting the initial value a(0)=6, we obtain the result shown above. This is the same as writing (3*2)^2^n but not the same as 3*2^2^n so therefore, the constant multiple of a1(n) is not also a solution.

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Question 3

> $rsolve(\{a(n) = 3\cdot a(n-1) - 2\cdot a(n-2), a(0) = 2, a(1) = 3\}, a(n));$

$$2^n + 1 \quad (13)$$

> # *Question* 4

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rsolve({a(n) = 2·a(n − 1) + 2·a(n − 2) − 2·a(n − 3), a(0) = 3, a(1) = 2, a(2) = 6}, a(n));
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$$\sum_{\_R = RootOf(2\_Z^3 - 2\_Z^2 - 2\_Z + 1)} \left(- \frac{(-4\_R^2 - 4\_R + 3)\left(\frac{1}{\_R}\right)^n}{(6\_R^2 - 4\_R - 2)\_R} \right) \quad (14)$$

> # *Question* 5

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rsolve({a(n) = a(n − 4), a(0) = 1, a(1) = 0, a(2) = 0, a(3) = 0}, a(n));
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$$\frac{1}{4} + \frac{(-1)^n}{4} + \frac{I^n}{4} + \frac{(-I)^n}{4} \quad (15)$$