```
#OK to post homework
#Shreya Ghosh, 9-13-2021, Assignment #2
#1i.
a := proc(n) option remember :
if n = 0 then 0 :
elif n = 1 then 1 :
elif n = 2 then 8 :
elif n = 3 then 27 :
else
4·a(n − 1) − 6·a(n − 2) + 4·a(n − 3) − a(n − 4) :
fi:
end:
seq(a(i), i = 1 ..8);
```

1, 8, 27, 64, 125, 216, 343, 512 **(1)**

```
#1ii. an=n^3
```

```
#1iii. Proof
```

$$\# \; n^3 = 4(n-1)^3 - 6(n-2)^3 + 4(n-3)^3 - (n-4)^3$$

$$\# \; n^3 = 4\left(n^3 - 3n^2 + 3n - 1\right) - 6\left(n^3 - 6n^2 + 12n - 8\right) + 4\left(n^3 - 9n^2 + 27n - 27\right) - \left(n^3 - 12n^2 + 48n - 64\right)$$

$$\# \; n^3 = 4n^3 - 12n^2 + 12n - 4 - 6n^3 + 36n^2 - 72n + 48 + 4n^3 - 36n^2 + 108n - 108 - n^3 + 12n^2 - 48n + 64$$

$$\# \; n^3 = n^3$$

```
#2.
```

$$\# \; \frac{dy}{dt} = \frac{y^3}{t+1}$$

$$\# \; int\frac{1}{y^3}dy = int\frac{1}{t+1}dt$$

$$\# \frac{-1}{2y^2} = \ln(t+1) + C$$

$$\# \frac{-1}{2(1)^2} = \ln(0+1) + C \rightarrow C = -\frac{1}{2}$$

$$\# \frac{-1}{2y^2} = \ln(t+1) - \frac{1}{2}$$

$$\# \; 2y^2 = -\frac{2}{2\ln(t+1) - 1}$$

$$\# \; y^2 = \frac{1}{1 - 2\ln(t+1)}$$

$$\# y = \frac{1}{\mathrm{sqrt}(1 - 2\ln(t+1))}$$

```
dsolve({D(y)(t) = y(t)^3/(t+1), y(0) = 1}, y(t));
```

$$y(t) = \frac{1}{\sqrt{1 - 2\ln(t+1)}} \quad \textbf{(2)}$$

```
> #3.
  # y''(t)-3y'(t)+2y(t)=0
  # r^2 - 3 r + 2 = 0 → r = 1, 2
  # y(t)=c1e^t + c2e^(2 t)
  # y'(t)=c1e^t + 2 c2e^t
  # 2=c1 +c2
  # 3 = c1 +2c2
  # c1 = 1, c2 = 1
  # y(t)=e^t + e^(2 t)
  dsolve({D(D(y))(t) - 3·D(y)(t) + 2·y(t) = 0, y(0) = 2, D(y)(0) = 3}, y(t));
```

$$y(t) = e^{t} + e^{2\,t} \qquad \textbf{(3)}$$

```
> with(LinearAlgebra);
```

[&x, Add, Adjoint, BackwardSubstitute, BandMatrix, Basis, BezoutMatrix, BidiagonalForm, **(4)**
BilinearForm, CARE, CharacteristicMatrix, CharacteristicPolynomial, Column,
ColumnDimension, ColumnOperation, ColumnSpace, CompanionMatrix,
CompressedSparseForm, ConditionNumber, ConstantMatrix, ConstantVector, Copy,
CreatePermutation, CrossProduct, DARE, DeleteColumn, DeleteRow, Determinant, Diagonal,
DiagonalMatrix, Dimension, Dimensions, DotProduct, EigenConditionNumbers, Eigenvalues,
Eigenvectors, Equal, ForwardSubstitute, FrobeniusForm, FromCompressedSparseForm,
FromSplitForm, GaussianElimination, GenerateEquations, GenerateMatrix, Generic,
GetResultDataType, GetResultShape, GivensRotationMatrix, GramSchmidt, HankelMatrix,
HermiteForm, HermitianTranspose, HessenbergForm, HilbertMatrix, HouseholderMatrix,
IdentityMatrix, IntersectionBasis, IsDefinite, IsOrthogonal, IsSimilar, IsUnitary,
JordanBlockMatrix, JordanForm, KroneckerProduct, LA_Main, LUDecomposition,
LeastSquares, LinearSolve, LyapunovSolve, Map, Map2, MatrixAdd, MatrixExponential,
MatrixFunction, MatrixInverse, MatrixMatrixMultiply, MatrixNorm, MatrixPower,
MatrixScalarMultiply, MatrixVectorMultiply, MinimalPolynomial, Minor, Modular, Multiply,
NoUserValue, Norm, Normalize, NullSpace, OuterProductMatrix, Permanent, Pivot,
PopovForm, ProjectionMatrix, QRDecomposition, RandomMatrix, RandomVector, Rank,
RationalCanonicalForm, ReducedRowEchelonForm, Row, RowDimension, RowOperation,
RowSpace, ScalarMatrix, ScalarMultiply, ScalarVector, SchurForm, SingularValues,
SmithForm, SplitForm, StronglyConnectedBlocks, SubMatrix, SubVector, SumBasis,
SylvesterMatrix, SylvesterSolve, ToeplitzMatrix, Trace, Transpose, TridiagonalForm,
UnitVector, VandermondeMatrix, VectorAdd, VectorAngle, VectorMatrixMultiply, VectorNorm,
VectorScalarMultiply, ZeroMatrix, ZeroVector, Zip]

```
> #4.
  # Characteristic Polynomial: r^2 - 6 r + 25 = 0 → r = 3 - 4 i, 3 + 4 i

    # 3-4i: A-(3-4i)I = [[4i, -4], [4, 4i]] -> RREF = [[1, i], [0, 0]] --> Av1 = [0,0] --> v1 = [-
    i, 1]
```

```
# 3+4i: A-(3+4i)I = [[-4i, -4], [4, -4i]] --> RREF = [[1, -i], [0, 0]] --> Av2 = [0, 0] -->
v2 = [i, 1]
A := Matrix([[3,-4], [4, 3]]);
```

$$A := \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix} \tag{5}$$

```
> evalf(Eigenvectors(A));
```

$$\begin{bmatrix} 3. + 4.\,\mathrm{I} \\ 3. - 4.\,\mathrm{I} \end{bmatrix}, \begin{bmatrix} \mathrm{I} & -\mathrm{I} \\ 1. & 1. \end{bmatrix} \tag{6}$$

```
>
```