Max Mekhanikov - HW 26 - Okay to post

P14) $x'(t) = 2x(t)(1-x(t))(2-x(t))(3-x(t))$

i. $F(x) = 2x(1-x)(2-x)(3-x) = 0$

$x=0, \ x=1, \ x=2, \ x=3$

P15) $x(n) = x(n-1)^3 + 2y(n-1)$

$y(n) = x(n-1)^2 + 5y(n-1)^2$

$F(x,y) = x^3 + 2y$

$G(x,y) = x^2 + 5y^2$

$x(0) = 1, \ y(0) = 3$

$x(1) = 1^3 + 2(3) = 7, \ y(1) = 1^2 + 5(3)^2 = 46$

$x(2) = 7^3 + 2(46) = 435, \ y(2) = 7^2 + 5(46)^2 = 10629$

$x(3) = 82334133, \ y(3) = 565067430$

$x(4) = x(3)^3 + 2(y(3)), \ y(4) = x(3)^2 + 5y(3)^2$

```
> read "/Users/maxmekhanikov/Downloads/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

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> # Max Mekhanikov - HW 26 - Okay to post
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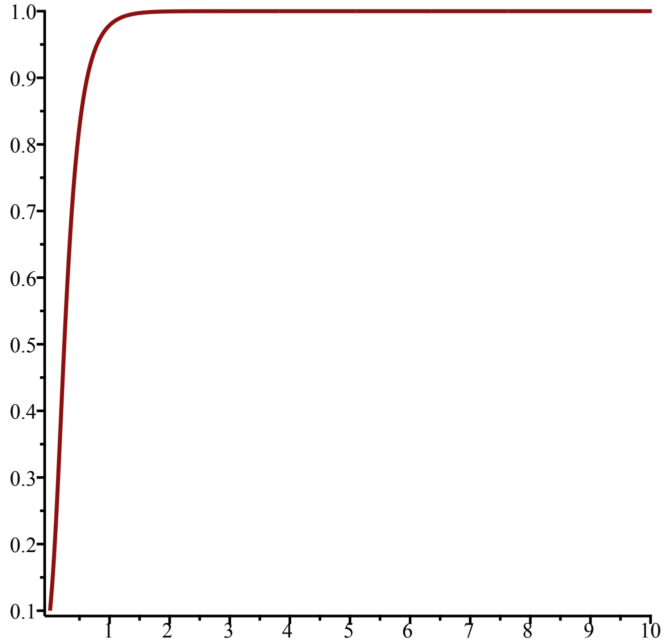
```
> Help(TimeSeries)
```

TimeSeries(F,x,pt,h,A,i): Inputs a transformation F in the list of variables x
The time-series of x[i] vs. time of the Dynamical system approximating the the autonomous continuous dynamical process
dx/dt=F(x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A
Try:

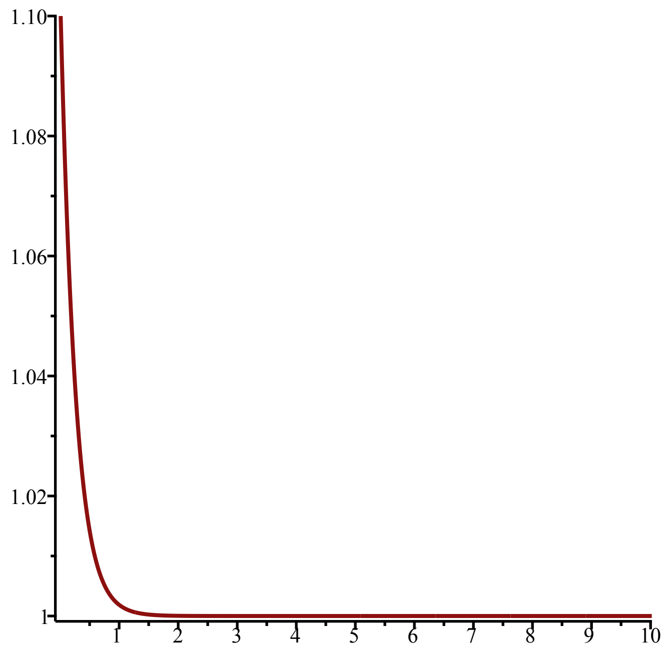
(2)

TimeSeries([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10,1);* **(2)**

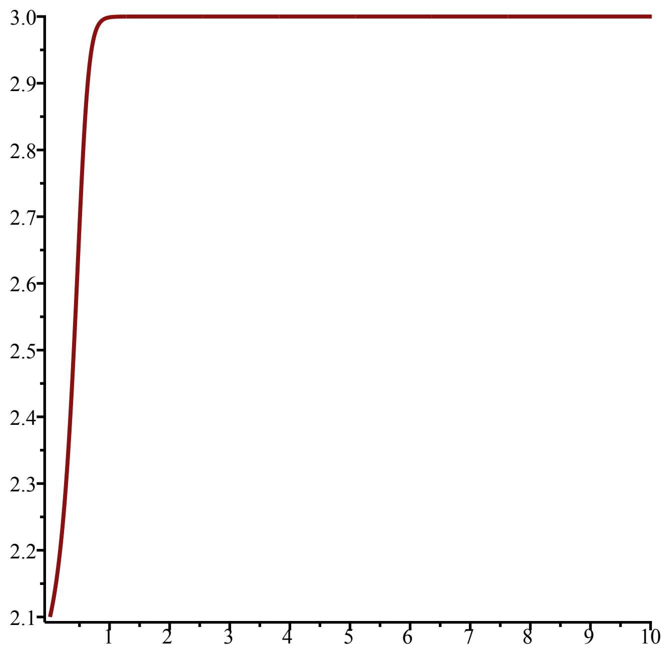
> $TimeSeries([2\cdot x\cdot(1-x)\cdot(2-x)\cdot(3-x)], [x], [0.1], 0.01, 10, 1)$



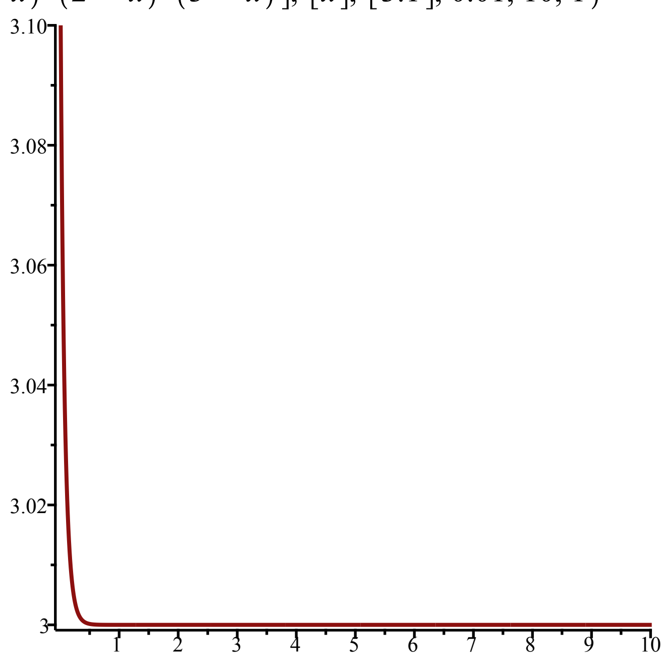
> $TimeSeries([2\cdot x\cdot(1-x)\cdot(2-x)\cdot(3-x)], [x], [1.1], 0.01, 10, 1)$



> $TimeSeries([2\cdot x\cdot(1-x)\cdot(2-x)\cdot(3-x)], [x], [2.1], 0.01, 10, 1)$



> $TimeSeries([2 \cdot x \cdot (1-x) \cdot (2-x) \cdot (3-x)], [x], [3.1], 0.01, 10, 1)$



> **# Starting near X=0 and X=1 both approach 1 in the long term behavior, while starting near X=2 and X=3 approach 3 long term as their asymptotes**
> **# iii.**

> *Help(EquP)*

EquP(F,x): Given a transformation F in the list of variables finds all the Equilibrium points of the continuous-time dynamical system x'(t)=F(x(t))

*EquP([5/2*x*(1-x)],[x]);*

EquP([y(1-x-y),x*(3-2*x-y)],[x,y]);* **(3)**

> $EquP([2 \cdot x \cdot (1-x) \cdot (2-x) \cdot (3-x)], [x])$

$$\{[0], [1], [2], [3]\} \quad (4)$$

> $SEquP([2 \cdot x \cdot (1-x) \cdot (2-x) \cdot (3-x)], [x])$

$$\{[1.], [3.]\} \quad (5)$$

```
> # SEquP uses the algorithm for determining whether or not a
  particular solution is stable. For continuous time functions such
  as this one, this is achieved by taking the derivative of the
  transformation function and plugging in each solution obtained from
  EquP. If the solutions result in F'(x) < 0, this means that
  particular value of X is stable. This is further supported by the
  results of the previous question as the only asymptotes were 1 and
  3, showing these were the only 2 stable equilibrium solutions.
>

  # P15
```

> $Help(Orb)$

Orb(F,x,x0,K1,K2): Inputs a transformation F in the list of variables x with initial point pt, outputs the trajectory of

of the discrete dynamical system (i.e. solutions of the difference equation): x(n)=F(x(n-1)) with x(0)= x0 from n=K1 to n=K2.

For the full trajectory (from n=0 to n=K2), use K1=0. Try:

*Orb(5/2*x*(1-x),[x], [0.5], 1000,1010);*

*Orb([(1+x+y)/(2+x+y),(6+x+y)/(2+4*x+5*y)],[x,y], [2.,3.], 1000,1010);* **(6)**

> $Orb([x^3 + 2 \cdot y, x^2 + 5 \cdot y^2], [x, y], [1, 3], 0, 3)$

[[1, 3], [7, 46], [435, 10629], [82334133, 565067430], [558135632816209194865497, 1603284911690886189]] **(7)**

```
> # P16
```

> $SFP\left(\left[\frac{(2+x+y)}{(2+2 \cdot x+2 \cdot y)}, \frac{(2+x+y)}{(1+2 \cdot x+2 \cdot y)}\right], [x, y]\right)$

$$\{[0.6953496364, 0.8641637014]\} \quad (8)$$

> $Orb\left(\left[\frac{(2+x+y)}{(2+2 \cdot x+2 \cdot y)}, \frac{(2+x+y)}{(1+2 \cdot x+2 \cdot y)}\right], [x, y], [0.5, 0.4], 1000, 1010\right)$

[[0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015]] **(9)**

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>
  # P17
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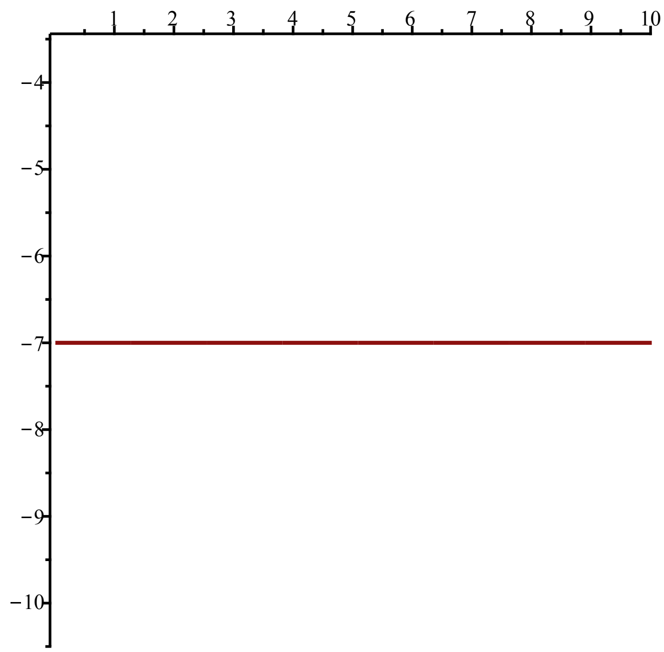
> $EquP([(1-2*x-3*y)*(2-2*x-3*y), (3-x-2*y)*(1-x-2*y)], [x, y]);$

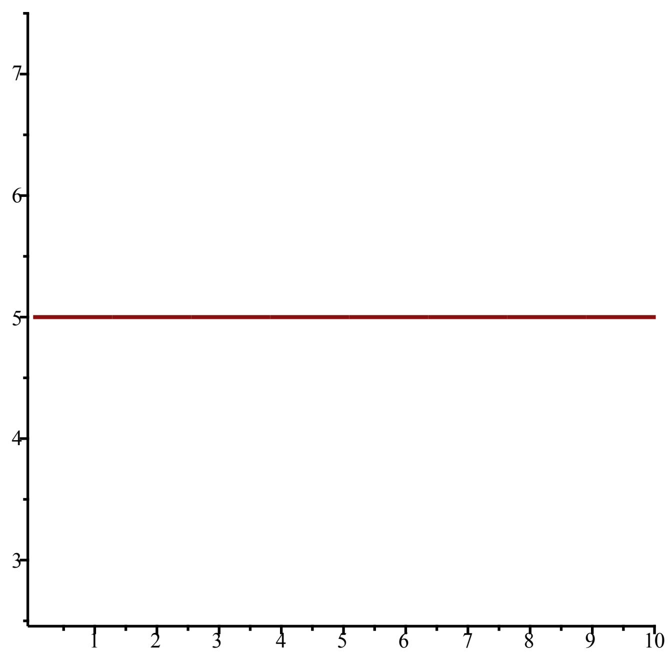
(10)

$$\{[-7, 5], [-5, 4], [-1, 1], [1, 0]\} \quad (10)$$

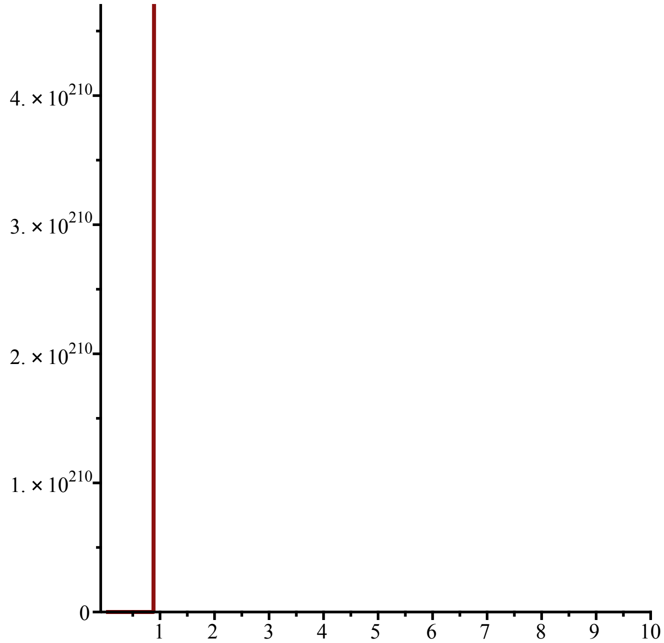
> *TimeSeries*([(1-2*x-3*y)*(2-2*x-3*y), (3-x-2*y)*(1-x-2*y)], [x, y], [-7, 5], 0.01, 10, 1);



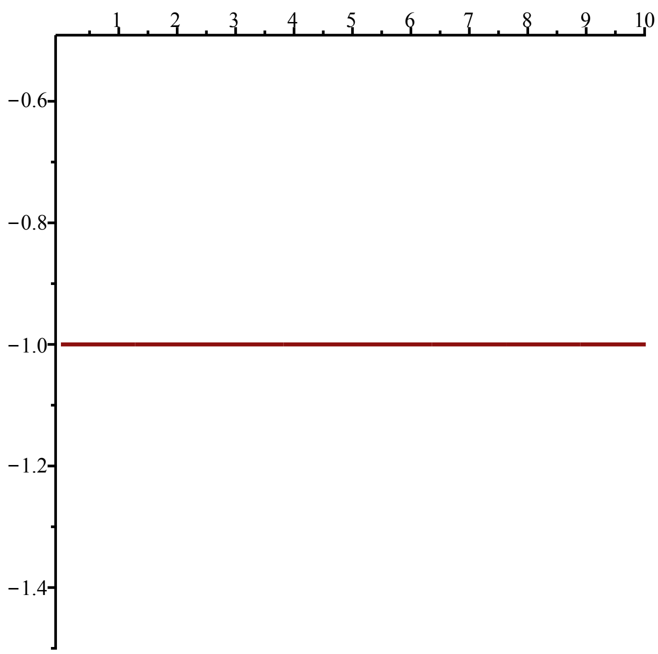
> *TimeSeries*([(1-2*x-3*y)*(2-2*x-3*y), (3-x-2*y)*(1-x-2*y)], [x, y], [-7, 5], 0.01, 10, 2);



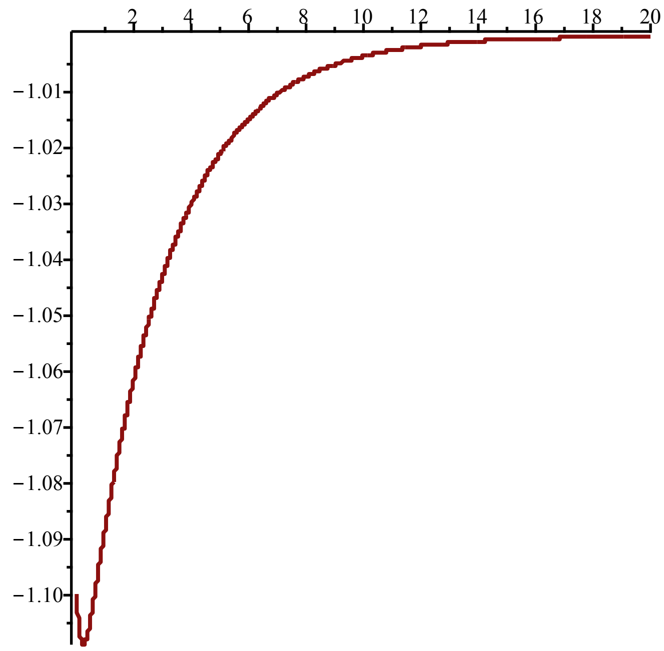
> *TimeSeries*([(1-2*x-3*y)*(2-2*x-3*y), (3-x-2*y)*(1-x-2*y)], [x, y], [-7.1, 5.1], 0.01, 10, 1);



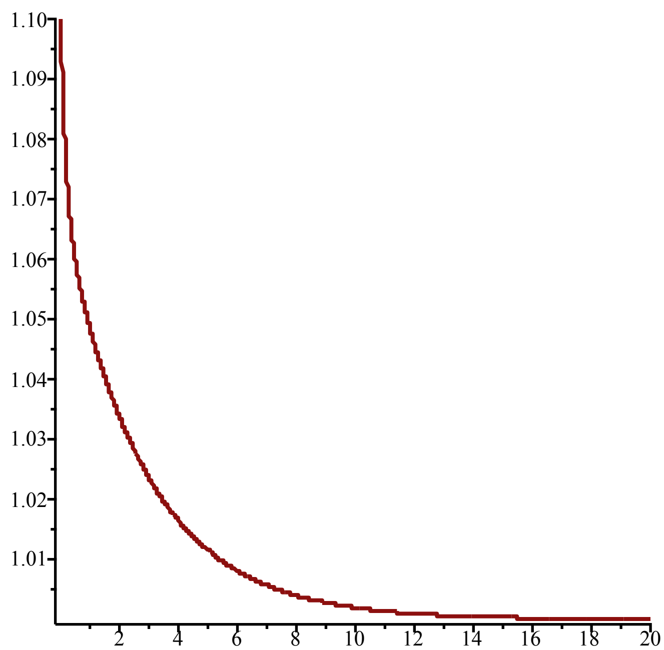
> *TimeSeries*([(1 - 2 * *x* - 3 * *y*) * (2 - 2 * *x* - 3 * *y*), (3 - *x* - 2 * *y*) * (1 - *x* - 2 * *y*)], [*x*, *y*], [-1, 1], 0.01, 10, 1);



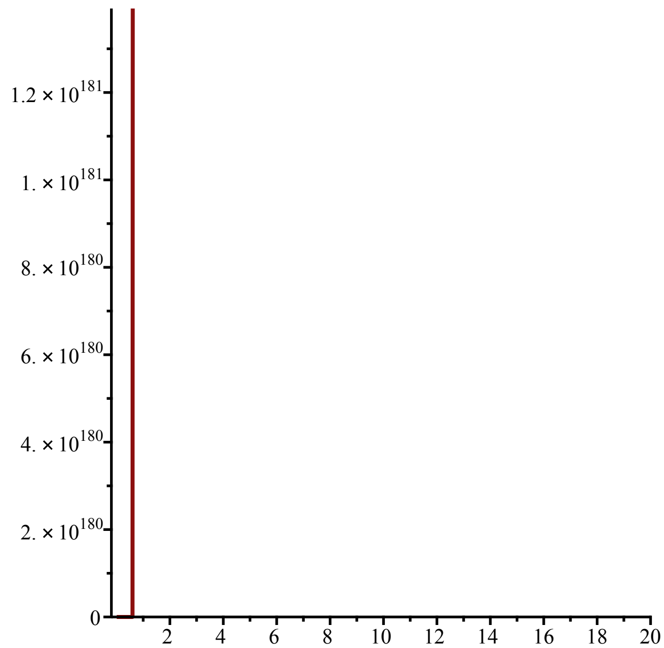
> *TimeSeries*([(1 - 2 * *x* - 3 * *y*) * (2 - 2 * *x* - 3 * *y*), (3 - *x* - 2 * *y*) * (1 - *x* - 2 * *y*)], [*x*, *y*], [-1.1, 1.1], 0.01, 20, 1);



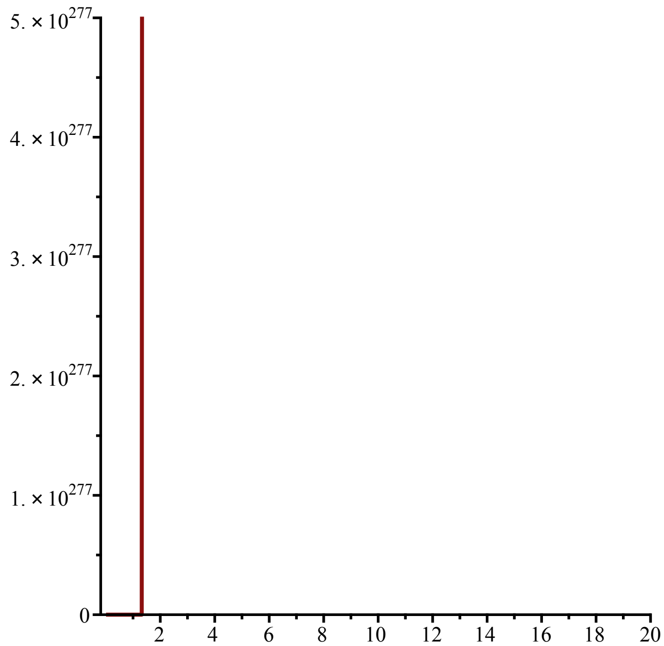
> *TimeSeries*([(1 − 2 * *x* − 3 * *y*) * (2 − 2 * *x* − 3 * *y*), (3 − *x* − 2 * *y*) * (1 − *x* − 2 * *y*)], [*x*, *y*], [−1.1, 1.1], 0.01, 20, 2);



> *TimeSeries*([(1 − 2 * *x* − 3 * *y*) * (2 − 2 * *x* − 3 * *y*), (3 − *x* − 2 * *y*) * (1 − *x* − 2 * *y*)], [*x*, *y*], [−5.1, 4.1], 0.01, 20, 2)



> *TimeSeries*$([(1-2*x-3*y)*(2-2*x-3*y), (3-x-2*y)*(1-x-2*y)], [x, y], [1.1, 0.1], 0.01, 20, 2)$



> # Clearly based of the graphs above, [-5,4] and [1,0] are both unstable as they both blow up when starting slightly away from the solution rather than reaching a stable long term equilibrium