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hw2b, 12/5/21

ok to Post

P14 i - Find all equilibrium solutions of the system

$$x'(t) = 2 \cdot x(t) \cdot (1 - x(t)) \cdot (2 - x(t)) \cdot (3 - x(t))$$

ii - Use Time series to investigate the stability of each equilibrium solutions (in .txt file)

iii - Confirm the stability of each equilibrium point using calculus

$$i - x'(t) = \underbrace{2 \cdot x(t) \cdot (1 - x(t)) \cdot (2 - x(t)) \cdot (3 - x(t))}_{f(x(t))} = -2x^4 + 12x^3 - 22x^2 + 12x$$

Note  $f'(x) = -8x^3 + 36x^2 - 44x + 12$

Let  $f(x) = 0$

$$0 = 2 \cdot x \cdot (1 - x) \cdot (2 - x) \cdot (3 - x) = -2x^4 + 12x^3 - 22x^2 + 12x$$

$x=0, x=1, x=2, x=3$  are equilibrium solutions

ii - in Maple

iii For an equilibrium to be stable,  $\frac{df}{dx}$  at the equilibrium solution must be less than 0, since this indicates that nearby values will approach the equilibrium solution from both sides.

$$f'(0) = 12 < 0 \times \underline{x=0 \text{ is unstable}}$$

$$f'(2) = 4 < 0 \times \underline{x=2 \text{ is unstable}}$$

$$f'(1) = -4 < 0 \checkmark \underline{x=1 \text{ is stable}}$$

$$f'(3) = -12 < 0 \checkmark \underline{x=3 \text{ is stable}}$$

P15 Find the first four terms of the Orbit by hand then confirm w/ Orb

$$x(n) = x(n-1)^3 + 2y(n-1)$$

$$y(n) = x(n-1)^2 + 5y(n-1)^2$$

$$x(0) = 1 \quad x(1) = 1^3 + 2 \cdot 3 = 7$$

$$y(0) = 3 \quad y(1) = 1^2 + 5 \cdot 3^2 = 46$$

$$x(2) = 7^3 + 2 \cdot 46 = 435$$

$$y(2) = 7^2 + 5 \cdot 46^2 = 10629$$

$$7^3 = 343$$

$$46^2 = 2116 \cdot 5 = 10580$$

$$x(3) = 435^3 + 2 \cdot 10629 = 82334133$$

$$y(3) = 435^2 + 5 \cdot 10629^2 = 565067430$$

$$\begin{array}{r} 435 \\ \cdot 435 \\ \hline 2175 \\ 13050 \\ 174000 \\ \hline 189225 \end{array}$$

$$\begin{array}{r} 189225 \\ \cdot 435 \\ \hline 7446125 \\ 5676750 \\ 75690000 \\ \hline 82312875 \\ + 21258 \\ \hline 82334133 \end{array}$$

$$\begin{array}{r} 10629 \quad 10629 \\ \cdot 10629 \\ \hline 95661 \\ 212580 \\ 6377400 \\ 0000000 \\ 00000000 \\ \hline 106290000 \end{array}$$

$$\begin{array}{r} 112975641.5 \\ + 564878205 \\ \hline 565067430 \end{array}$$

Orbit =  $((1,3), (7,46), (435, 10629), (82334133, 565067430))$

P16 & P17 on .txt file