```
> #OK to post
  #Julian Herman, 5th December, 2021, Assignment 26
> read `/Users/julianherman/Documents/Rutgers/Fall 2021/Dynamical Models In
      Biology/HW/DMB.txt` :
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #11')
> Orb([x^2 - 2·x + 2], [x], [2.0], 1000, 1010)
```

[[2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.]] **(2)**

```
> Orb([x^2 - 2·x + 2], [x], [2.01], 1000, 1010)
```

[[Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [**(3)**
Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [

Float(undefined)], [Float(undefined)], [Float(undefined)]]

> #x(n)=2 is an UNSTABLE equilibrium solution

>

> $Orb([x^2 - 2\cdot x + 2], [x], [1.0], 1000, 1010)$

[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000]] **(4)**

> $Orb([x^2 - 2\cdot x + 2], [x], [1.01], 1000, 1010)$

[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000]] **(5)**

> $Orb([x^2 - 2\cdot x + 2], [x], [1.5], 1000, 1010)$

[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000]] **(6)**

> #x(n)=1 is a STABLE equilibrium solution

>

> #12')

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.0], 1000, 1010\right)$

[[0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.]] **(7)**

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.01], 1000, 1010\right)$

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(8)**

> #x(n)=0 is an UNSTABLE equilibrium solution

>

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.6], 1000, 1010\right)$

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(9)**

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.61], 1000, 1010\right)$

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(10)**

> $Orb\left(\left[\frac{5}{2}\cdot x\cdot(1-x)\right], [x], [0.62], 1000, 1010\right)$

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(11)**

> *#x(n)=* $\frac{3}{5}$ = 0.6 *is STABLE equilibrium solution*

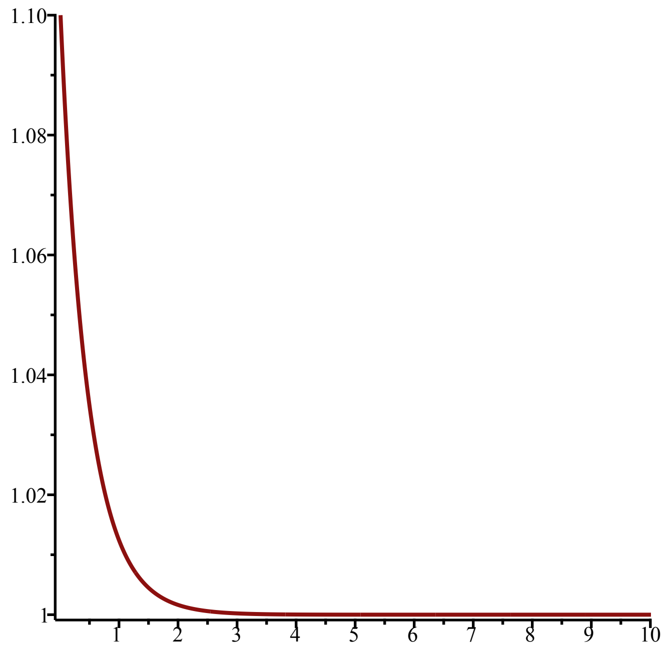
>

> *#Random examples)*

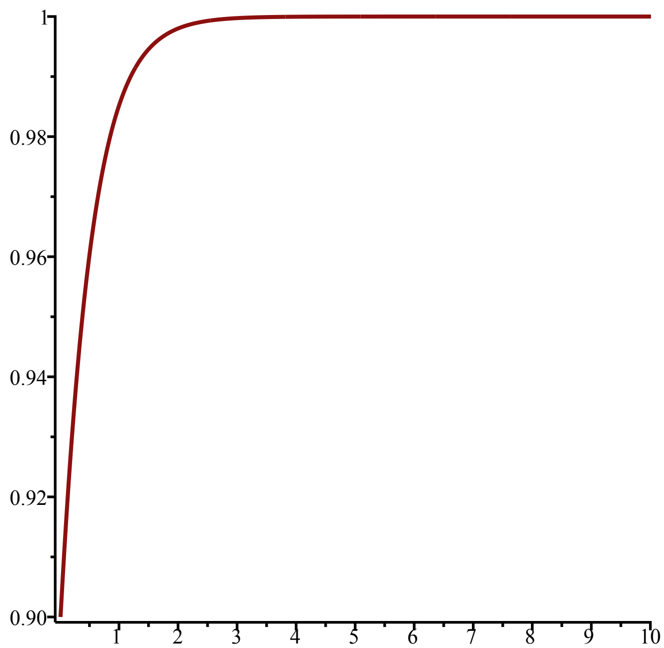
> $dsolve(\{x'(t) = 2\cdot x(t)\cdot(1-x(t)), x(0) = 0.9\}, x(t))$

$$x(t) = \frac{9}{9 + e^{-2t}} \quad \textbf{(12)}$$

> $TimeSeries([2\cdot x\cdot(1-x)], [x], [1.1], 0.01, 10, 1)$



> $TimeSeries([2\cdot x\cdot(1-x)], [x], [0.9], 0.01, 10, 1)$



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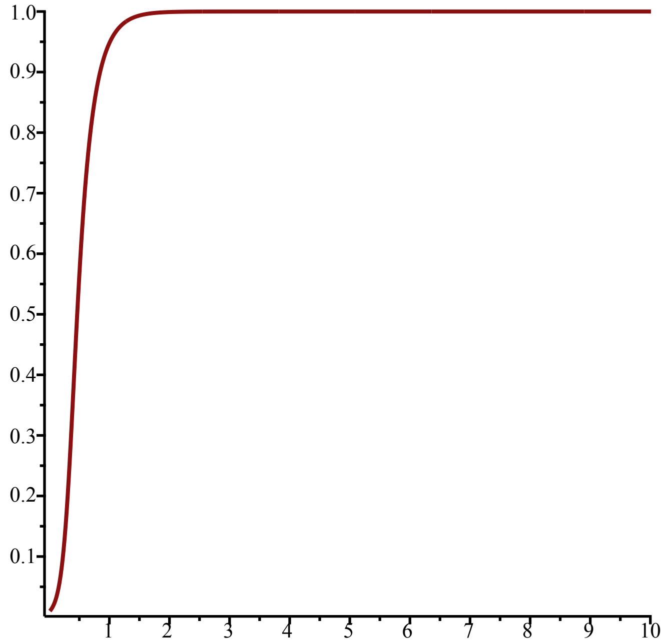
> *Help*(*TimeSeries*)

TimeSeries(F,x,pt,h,A,i): Inputs a transformation F in the list of variables x
The time-series of x[i] vs. time of the Dynamical system approximating the the autonomous continuous dynamical process
dx/dt=F(x(t)) by a discrete time dynamical system with step-size h from t=0 to t=A
Try:
TimeSeries([x(1-y),y*(1-x)],[x,y],[0.5,0.5], 0.01, 10,1);* **(13)**

> #14)

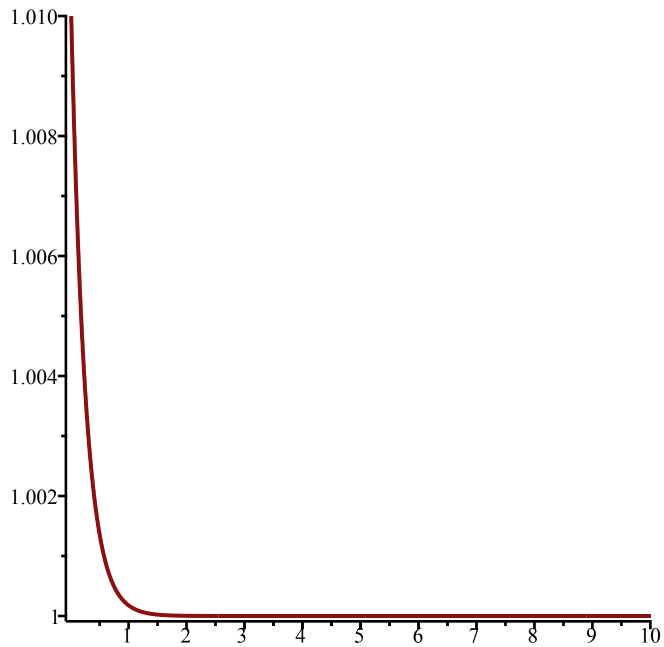
> #ii)

> *TimeSeries*$([2\cdot x\cdot(1-x)\cdot(2-x)\cdot(3-x)], [x], [0.01], 0.01, 10, 1)$



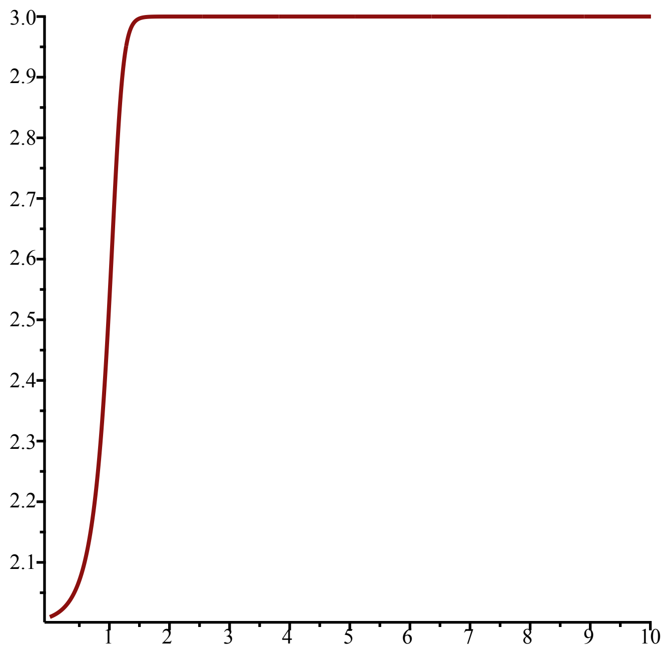
> *#x(t)=0 appears UNSTABLE: a small change and it approaches the solution x(t)=1*

> $TimeSeries([2 \cdot x \cdot (1 - x) \cdot (2 - x) \cdot (3 - x)], [x], [1.01], 0.01, 10, 1)$



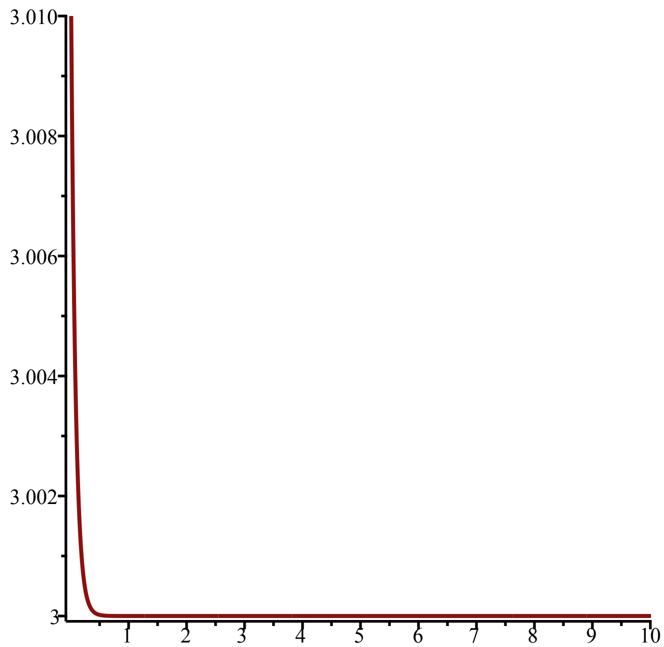
> *#x(t)=1 appears STABLE: a small change and it converges back to itself:* $x(t) = 1$

> $TimeSeries([2 \cdot x \cdot (1 - x) \cdot (2 - x) \cdot (3 - x)], [x], [2.01], 0.01, 10, 1)$



> *#x(t)=2 appears UNSTABLE: a small change and it approaches the solution x(t)=3*

> $TimeSeries([2 \cdot x \cdot (1-x) \cdot (2-x) \cdot (3-x)], [x], [3.01], 0.01, 10, 1)$



> *#x(t)=3 appears STABLE: a small change and it converges back to itself: x(t)=3*

>

> *#iii)*

> $expand(2 \cdot x \cdot (1-x) \cdot (2-x) \cdot (3-x))$

$$-2x^4 + 12x^3 - 22x^2 + 12x \quad \textbf{(14)}$$

> $f(x) := -2x^4 + 12x^3 - 22x^2 + 12x$

$$f := x \mapsto -2 \cdot x^4 + 12 \cdot x^3 - 22 \cdot x^2 + 12 \cdot x \quad \textbf{(15)}$$

> $f'(x)$

(16)

$$-8x^3 + 36x^2 - 44x + 12 \quad (16)$$

> $subs(x = 2, f'(x))$

$$4 \quad (17)$$

> $subs(x = 3, f'(x))$

$$-12 \quad (18)$$

>

>

>

> #15)

> $Orb([x^3 + 2\cdot y, x^2 + 5\cdot y^2], [x, y], [1, 3], 0, 3)$

$$[[1, 3], [7, 46], [435, 10629], [82334133, 565067430]] \quad (19)$$

>

>

> #16)

> $F := \left[\frac{2+x+y}{2+2\cdot x+2\cdot y}, \frac{2+x+y}{1+2\cdot x+2\cdot y}\right]$

$$F := \left[\frac{2+x+y}{2+2x+2y}, \frac{2+x+y}{1+2x+2y}\right] \quad (20)$$

> $SFP(F, [x, y])$

$$\{[0.6953496364, 0.8641637014]\} \quad (21)$$

> $Orb(F, [x, y], [0.5, 0.4], 1000, 1010)$

[[0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010]] **(22)**

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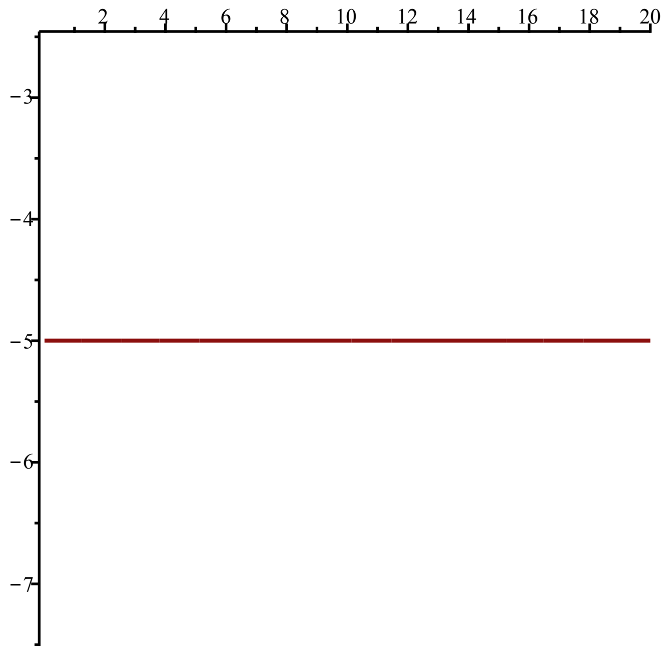
>

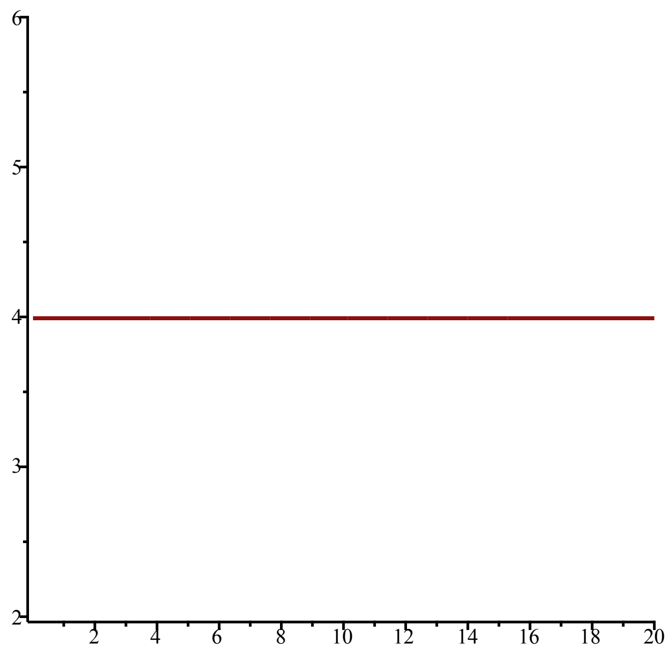
>

> #17)

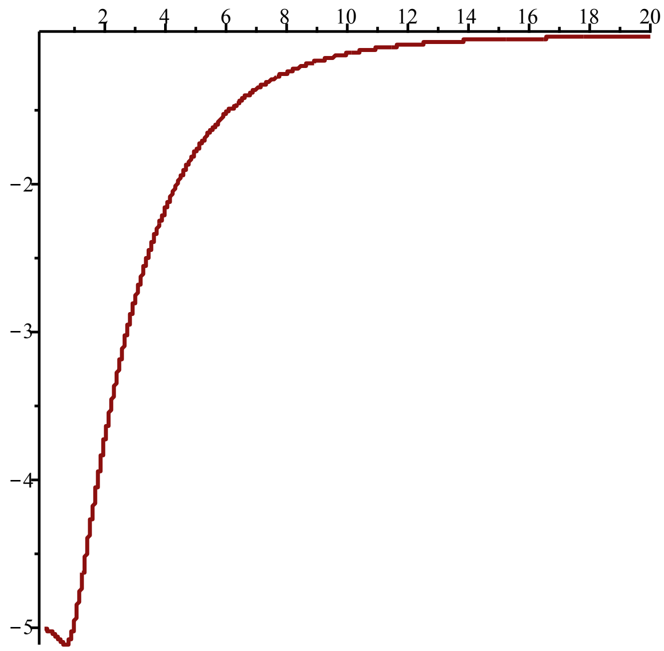
> TimeSeries([(1-2*x-3*y)*(2-2*x-3*y), (3-x-2*y)*(1-x-2*y)], [x, y], [-5.0, 4.0], 0.01, 20, 1)



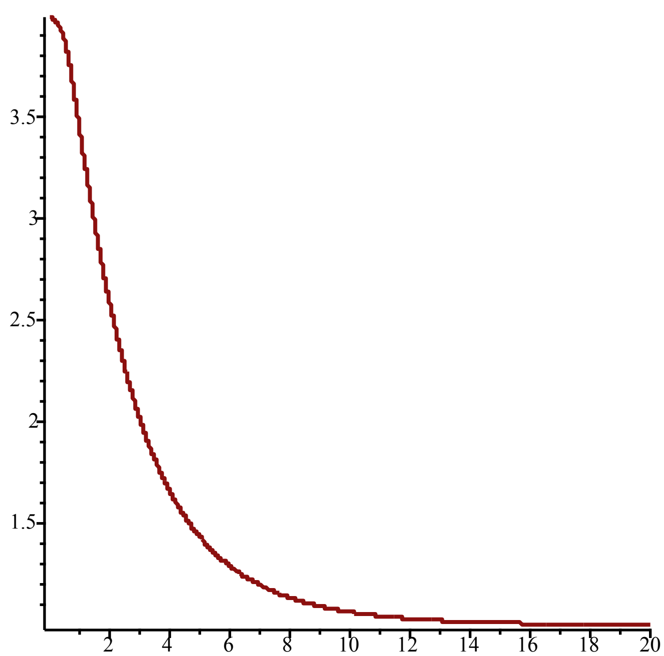
> *TimeSeries*([(1 − 2 * *x* − 3 * *y*) * (2 − 2 * *x* − 3 * *y*), (3 − *x* − 2 * *y*) * (1 − *x* − 2 * *y*)], [*x*, *y*], [−5.0, 4.0], 0.01, 20, 2)



> *TimeSeries*([(1 − 2 * *x* − 3 * *y*) * (2 − 2 * *x* − 3 * *y*), (3 − *x* − 2 * *y*) * (1 − *x* − 2 * *y*)], [*x*, *y*], [−5.01, 3.99], 0.01, 20, 1)



> *TimeSeries*([(1 − 2 * *x* − 3 * *y*) * (2 − 2 * *x* − 3 * *y*), (3 − *x* − 2 * *y*) * (1 − *x* − 2 * *y*)], [*x*, *y*], [−5.01, 3.99], 0.01, 20, 2)

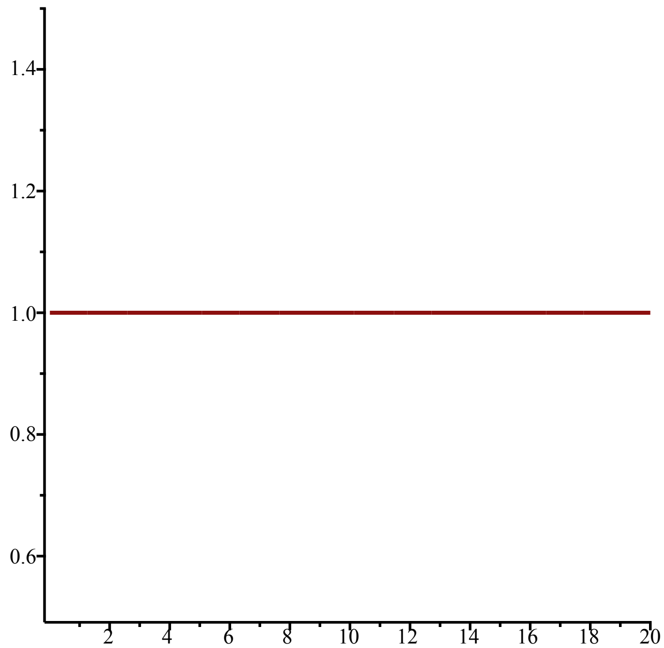


> *#Clearly, the EQ solution [-5,4] is unstable, a small change to the initial conditions: [-5.01,3.99] and the system drifts towards the other STABLE EQ solution [-1,1].*

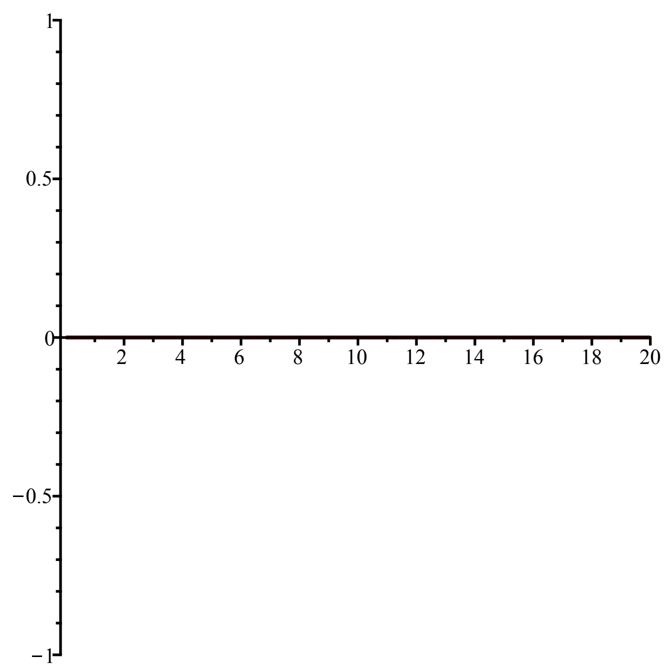
>

>

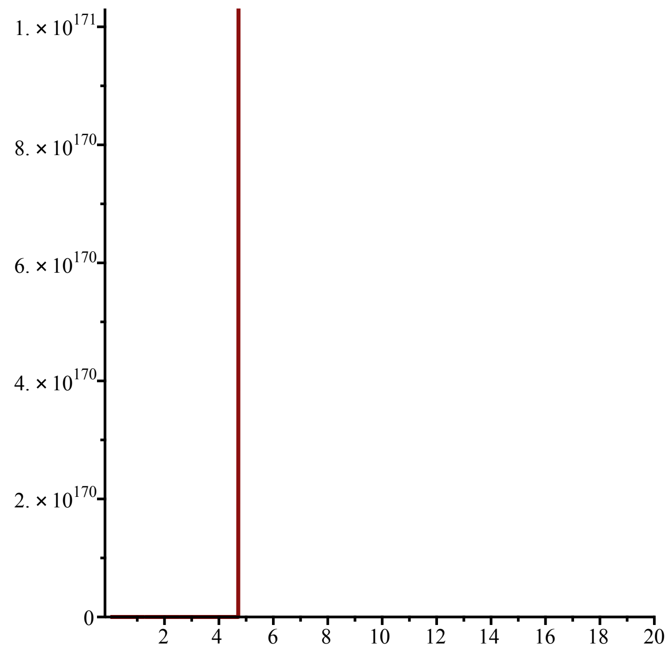
> *TimeSeries*([(1 − 2 * *x* − 3 * *y*) * (2 − 2 * *x* − 3 * *y*), (3 − *x* − 2 * *y*) * (1 − *x* − 2 * *y*)], [*x*, *y*], [1.0, 0.0], 0.01, 20, 1)



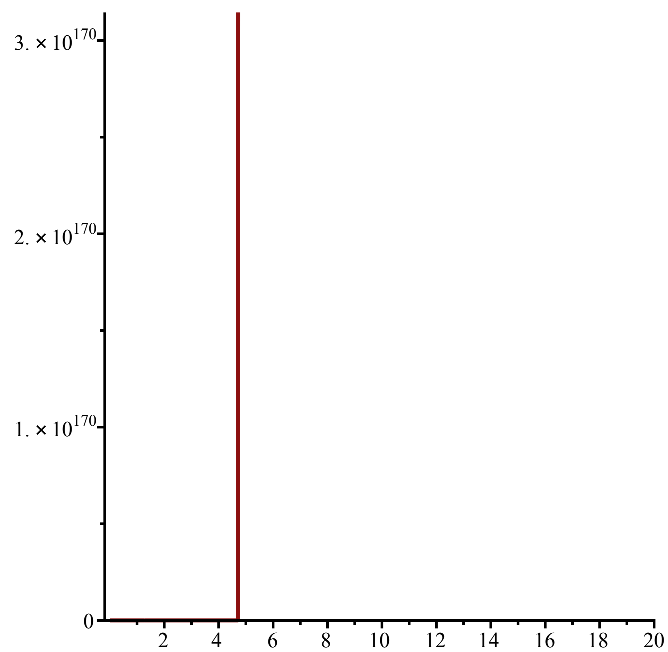
> *TimeSeries*([(1−2*x−3*y)*(2−2*x−3*y), (3−x−2*y)*(1−x−2*y)], [x, y], [1.0, 0.0], 0.01, 20, 2)



> *TimeSeries*([(1−2*x−3*y)*(2−2*x−3*y), (3−x−2*y)*(1−x−2*y)], [x, y], [1.01, 0.01], 0.01, 20, 1)



> *TimeSeries*([(1 - 2 * *x* - 3 * *y*) * (2 - 2 * *x* - 3 * *y*), (3 - *x* - 2 * *y*) * (1 - *x* - 2 * *y*)], [*x*, *y*], [1.01, 0.01], 0.01, 20, 2)



> *#Clearly the EQ solution [1,0] is UNSTABLE: a small change to initial conditions [1.01,0.01] and the system blows-up to [infinity, infinity]*

11") Underlying function: $f(x) = x^2 - 2x + 2$

$f'(x) = 2x - 2$

For $x(n) = 2$: $f'(2) = 2(2) - 2 = 2$

$|2| = 2 \not< 1$ **UNSTABLE**

For $x(n) = 1$: $f'(1) = 2(1) - 2 = 0$

$|0| = 0 < 1$ ✓ **STABLE**

12") Underlying function: $f(x) = \frac{5}{2}x \cdot (1-x) = \frac{5}{2}x - \frac{5}{2}x^2$

$f'(x) = \frac{5}{2} - 5x$

For $x(n) = 0$: $f'(0) = \frac{5}{2} - 5(0) = \frac{5}{2} = 2.5$

$|2.5| = 2.5 \not< 1$ **UNSTABLE**

For $x(n) = \frac{3}{5}$: $f'\left(\frac{3}{5}\right) = \frac{5}{2} - 5\left(\frac{3}{5}\right) = 2.5 - 3 = -0.5$

$|-0.5| = 0.5 < 1$ ✓ **STABLE**

14.) $x'(t) = 2 \cdot x(t)\,(1 - x(t))(2 - x(t))(3 - x(t))$

i.) Underlying function: $f(x) = 2x(1-x)(2-x)(3-x)$

$f(x) = 0$: $2x(1-x)(2-x)(3-x) = 0$

E.Q. solutions: $x(t)=0,\ x(t)=1,\ x(t)=2,\ x(t)=3$

ii) Refer to Maple

iii) To conclusively decide if the E.Q. solutions are stable or not, we must check if $f'(x(t)=c) < 0$ (if it is less than 0, $x(t)=c$ is STABLE). This is because the jacobian matrix, in this case, is a 1×1 matrix consisting solely of the derivative of the underlying function: $f'(x)$, so if the eignvalue (which, at $x(t)=c$ is $f'(c)$) is negative, it is stable.

$f(x) = 2x(1-x)(2-x)(3-x) = -2x^4 + 12x^3 - 22x^2 + 12x$

$f'(x) = -8x^3 + 36x^2 - 44x + 12$

For: $x(t)=0$: $f'(0) = 12 \not< 0$ UNSTABLE

$x(t)=1$: $f'(1) = -8 + 36 - 44 + 12 = -4 < 0$ ✓ STABLE

$x(t)=2$: $f'(2) = 4 \not< 0$ UNSTABLE

$x(t)=3$: $f'(3) = -12 < 0$ ✓ STABLE

15.) $x(n) = x(n-1)^3 + 2y(n-1)$, $x(0) = 1$

$y(n) = x(n-1)^2 + 5y(n-1)^2$, $y(0) = 3$

$x(1) = x(0)^3 + 2 \cdot y(0) = 1^3 + 2 \cdot 3 = 7$

$y(1) = x(0)^2 + 5 \cdot y(0)^2 = 1^2 + 5 \cdot 3^2 = 46$

$x(2) = x(1)^3 + 2 \cdot y(1) = 7^3 + 2 \cdot 46 = 435$

$y(2) = x(1)^2 + 5 \cdot y(1)^2 = 7^2 + 5 \cdot 46^2 = 10629$

$x(3) = x(2)^3 + 2 \cdot y(2) = 435^3 + 2 \cdot 10629 = 82334133$

$y(3) = x(2)^2 + 5 \cdot y(2)^2 = 435^2 + 5 \cdot 10629^2 =$
565067430

[[1,3], [7,46], [435, 10629], [82334133, 565067430]]