Homework 26
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```
> read `C:/Users/cgrie/Dynam Models Bio/Homeworks/HW24/DMB.txt`
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

PROBLEM 14

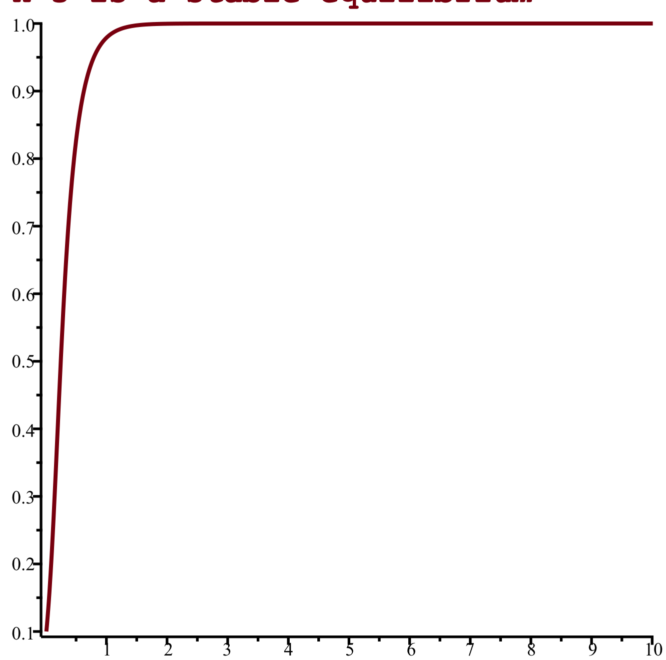
(i)
For the CONTINUOUS equation $x'(t) = 2\,x(t)\,(1 - x(t))\,(2 - x(t))\,(3 - x(t))$
The underlying transformation is: $f(x) = 2\,x(1 - x)\,(2 - x)\,(3 - x)$
Which has equilibrium solutions
$x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 3$
because $f(x) = 0$ is the necessary condition for an equilibrium in a continuous case

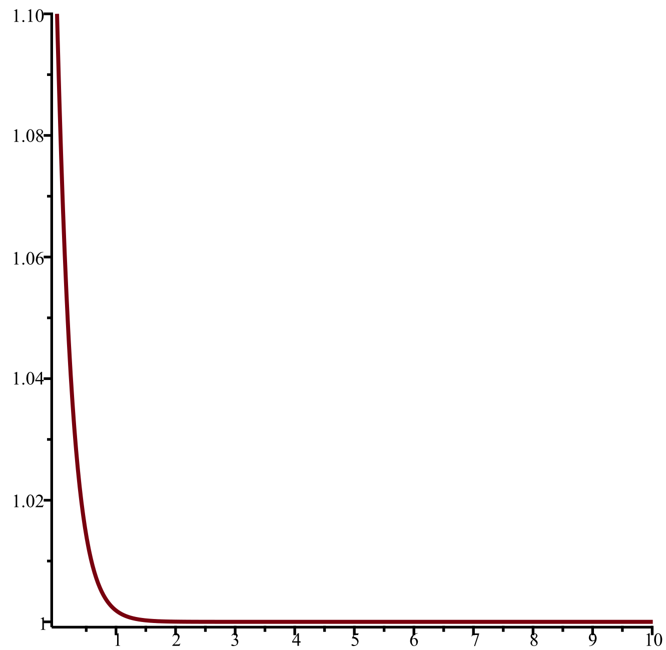
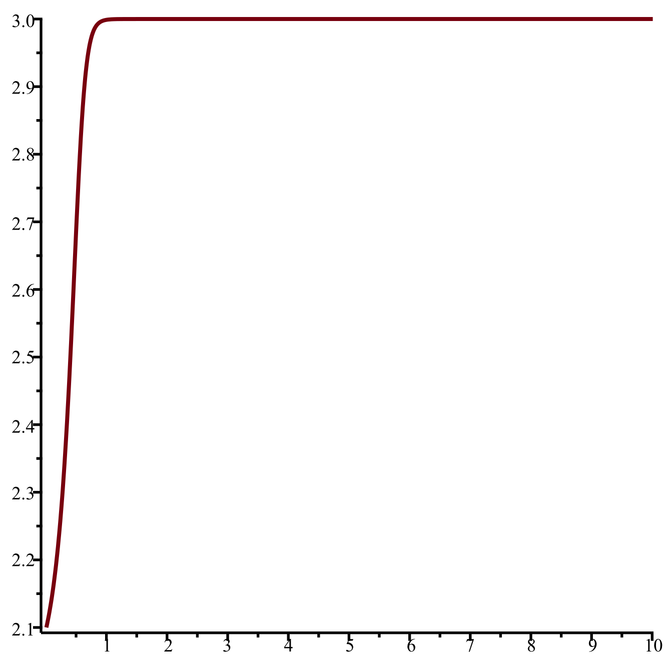
(ii) Use timeSeries to determine which equilibrium is stable

Sol: to do this, choose initial conditions in between the equilibrium solutions

$$F\_list := [\ ] \qquad (2)$$

```
> TimeSeries([2*x*(1-x)*(2-x)*(3-x)],[x],[0.1],0.01,10,1);
  TimeSeries([2*x*(1-x)*(2-x)*(3-x)],[x],[1.1],0.01,10,1);
  #The above 2 plots confirm x=1 is a stable equilibrium
  TimeSeries([2*x*(1-x)*(2-x)*(3-x)],[x],[2.1],0.01,10,1);
  #The plot above and the plot immediately above the previous
  comment confirm x=2 is an unstable equilibrium solution
  TimeSeries([2*x*(1-x)*(2-x)*(3-x)],[x],[3.1],0.01,10,1);
  #The plot above and the plot immediately above the previous
  comment confirm x=3 is a stable equilibrium
```




1.10
1.08
1.06
1.04
1.02
1
1
2
3
4
5
6
7
8
9
10

3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
1
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3
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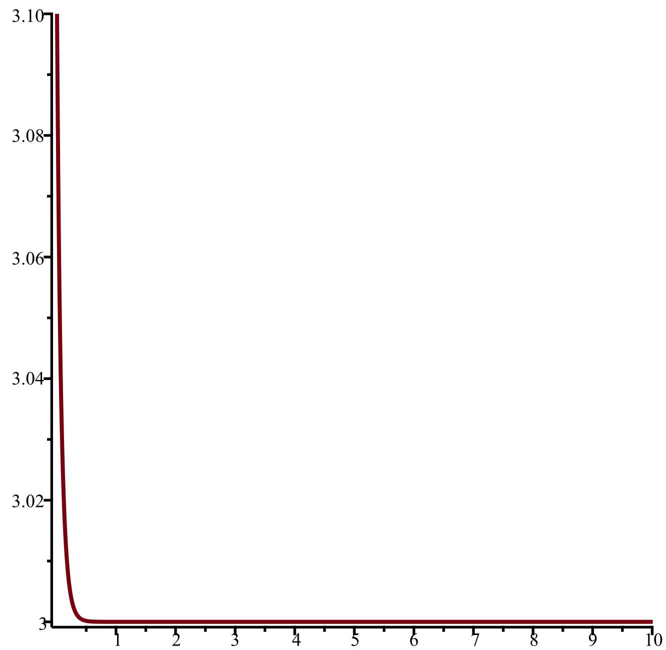

Question P15

```
> Orb([x^3+2*y,x^2+5*y^2],[x,y],[1.,3.],0,5);
```

$$[[1., 3.], [7., 46.], [435., 10629.], [8.2334133 \times 10^{7}, 5.65067430 \times 10^{8}], [5.581356328 \times 10^{23}, 1.603284911 \times 10^{18}], [1.738678363 \times 10^{71}, 3.115153846 \times 10^{47}]] \quad (3)$$

Confirmed!

Question P16

```
> UT := [(2+x+y)/(2+2*x+2*y),(2+x+y)/(1+2*x+2*y)]
```

$$UT := \left[\frac{2+x+y}{2+2\,x+2\,y}, \frac{2+x+y}{1+2\,x+2\,y}\right] \quad (4)$$

SFP command:

```
> sfp_p16 := SFP(UT,[x,y]);
```

$$sfp\_p16 := \{[0.6953496364, 0.8641637014]\} \quad (5)$$

The numbers match up with bc.pdf

```
> sfp_p16[1]
```

$$[0.6953496364, 0.8641637014] \quad (6)$$

Orb command:

```
> Orb(UT,[x,y],[0.6953496364, 0.8641637014],1000,1100);
```

[[0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, **(7)**
0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015],
[0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365,
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[0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362,
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0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013],
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[0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362,
0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013],
[0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364,
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[0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362,
0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013],
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[0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365,
0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010],
[0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362,
0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013],

[0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015], [0.6953496364, 0.8641637013], [0.6953496362, 0.8641637010], [0.6953496365, 0.8641637015]]

#yes the trajectory indicates that numerically,

P17

For the continuous time dynamical system

x'(t) = (1 - 2x(t) - 3y(t)) (2 - 2x(t) - 3y(t))
y'(t) = (3 - x(t) - 2y(t)) (1 - x(t) - 2y(t))

The underlying transformation is:
[(1 - 2x - 3y) (2 - 2x - 3y),(3 - x - 2y) (1 - x - 2y)]

CONVINCE yourself that the equilibrium solutions [-5,4] and [1,0] are unstable

```
> print(JAC);
```

```
proc(F, x)                                                        (8)
    local i, j;
    if not (type(F, list) and type(x, list) and nops(F) = nops(x)) then
        print(`Bad input`); RETURN(FAIL)
    end if;
    normal([seq([seq(diff(F[i], x[j]), j = 1 ..nops(x))], i = 1 ..nops(F))])
end proc
```

```
> jac := JAC([(1 - 2*x - 3*y) *(2 - 2*x - 3*y),(3 - x - 2*y) (1 - x
  - 2*y)],[x,y]);
  jac_54 := subs({x=-5,y=-4},jac);
  #Since D evaluates to 0 because derivative of a constant term is
  0,
  jac_54_final := Matrix([[-94,-141],[0,0]]);
  evalf(Eigenvalues(jac_54_final));

  #Because one of the eigenvalues is non-negative (the eigenvalue
  that has a value of 0), the equilibrium [-5,4] is unstable
```

$jac := [[-6 + 8x + 12y, -9 + 12x + 18y], [D(x)(1 - x - 2y) + 2 D(y)(1 - x - 2y), 2 D(x)(1 - x - 2y) + 4 D(y)(1 - x - 2y)]]$

$jac\_54 := [[-94, -141], [D(-5)(14) + 2 D(-4)(14), 2 D(-5)(14) + 4 D(-4)(14)]]$

$$jac\_54\_final := \begin{bmatrix} -94 & -141 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ -94. \end{bmatrix} \quad (9)$$

```
> jac_10 := subs({x=1,y=0},jac);
  #Thus
  jac_10_final := Matrix([[2,3],[0,0]]);
  evalf(Eigenvalues(jac_10_final));
  #We see that the equilibrium solution [1,0] is unstable because
  at least one of the eigenvalues of its jacobian is nonzero
```

$$jac\_10 := [[2, 3], [D(1)(0) + 2\,D(0)(0), 2\,D(1)(0) + 4\,D(0)(0)]]$$

$$jac\_10\_final := \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0. \\ 2. \end{bmatrix} \quad (10)$$

```
> 435^2+5*10629^2
```

$$565067430 \quad (11)$$

Homework 2

P15: By hand, find the first four terms of the orbit, starting at $n=0$ of the discrete time dynamical system,

if $x(0) = 1$, $y(0) = 3$. Confirm it with the output of orb

$x(n) = x(n-1)^3 + 2y(n-1)$

$y(n) = x(n-1)^2 + 5y(n-1)^2$

FIRST Term: $x(0) = 1$, $y(0) = 3$

SECOND TERM:

$x(1) = (1)^3 + 2(3) = 7$

$y(1) = (1)^2 + 5(3)^2 = 46$

THIRD TERM:

$x(2) = (7)^3 + 2(46) = 343 + 92 = 435$

$y(2) = (7)^2 + 5(46)^2 = 10629$

FOURTH TERM:

$x(3) = (435)^3 + 2(10629) = 8.2334133$

$y(3) = (435)^2 + 5(10629)^2 = 565067430$