*OK to post Homework
* Jeton Hida, Assignment 25, December 2, 2021

P1: $z^3 + 3z^2 - 11z + 2 = 0$ Is $z=2$ a solution?
$(2)^3 + 3(2)^2 - 11(2) + 2 = 0$
$8 + 12 - 22 + 2 = 0 \to 0 = 0$ Yes $z=2$ is a solution!
Is $z=3$ also a solution?
$(3)^3 + 3(3)^2 - 11(3) + 2 = 0$
$27 + 27 - 33 + 2 = 0$
$21 + 2 = 0$ X No $z=3$ is not a solution.

P2: $\sin z = 0$ Is $z = \pi$ a solution?
$\sin \pi = 0 \to 0 = 0$ Yes $z = \pi$ is a solution
What about $z = \pi/2$?
$\sin \pi/2 = 0 \to 1 \neq 0$ No $z = \pi/2$ is NOT a solution

P3: $\sin^2 z + \cos^2 z = 1$ Is $z = \pi/3$ a solution
$\sin^2(\pi/3) + \cos^2(\pi/3) = 1$
$.75 + .25 = 1 \to 1 = 1$ ✓ Yes $z = \pi/3$ is a solution
How about $z = \pi/5$
$\sin^2(\pi/5) + \cos^2(\pi/5) = 1$
$.3455 + .6545 = 1 \to 1 = 1$ Yes $z = \pi/5$ is also a solution

P4: $\sin^2 z + \cos^2 z = 1$
This a famous pythagorean identity, thus it is always true
The set of all solutions is $\mathbb{R}$ (set of all real numbers)

P5: $x(t) = t^4$
Rate of Change $x'(t) = 4t^3 \to x'(2) = 4(2)^3 = 32$
Rate of Change of Rate of Change $x''(t) = 12t^2 \to x''(2) = 12(2)^2 = 48$

P6: $f(x) = (x-1)(x-2)(x-3) + x$ Are $x = 1, 2, 3, -1$ fixed points?

$f(1) = (1-1)(1-2)(1-3) + 1$

$f(1) = 0 + 1 \Rightarrow f(1) = 1$ ✓

$f(2) = (2-1)(2-2)(2-3) + 2$

$f(2) = 0 + 2 \Rightarrow f(2) = 2$ ✓

$f(3) = (3-1)(3-2)(3-3) + 3$

$f(3) = 0 + 3 \Rightarrow f(3) = 3$ ✓

$f(-1) = (-1-1)(-1-2)(-1-3) + -1$

$f(-1) = (-2)(-3)(-4) + -1 \Rightarrow f(-1) = -25$ x

$x=1, x=2, x=3$ are fixed points

$x=-1$ is NOT a fixed point

P7: $f(x,y) = (x+y+1, x-y-2)$ Is $(x,y) = (0,-1)$ a fixed point?

$f(0,-1) = (0+-1+1, 0+1-2)$

$f(0,-1) = (0, -1)$ Yes $(x,y) = (0,-1)$ is a fixed point

Is $(x,y) = (1,1)$ also a fixed point?

$f(1,1) = (1+1+1, 1-1-2)$

$f(1,1) = (3, -2)$ No $(x,y) = (1,1)$ is not a fixed point

P8: $f(x) = 1/(x+1)$

(i) $x(0) = 0.5$ $x(1) = 1/(.5+1) = 1/1.5 = .6667$ $x(2) = 1/(.6667+1) = 1/1.6667 = .6$

(ii) Orb([1/(x+1)], [x], [0.5], 0, 2);

(iii) Orb([1/(x+1)], [x], [0.5], 1000, 1000)[1];

You get [.6180339887]

P9: $f(x,y,z) = (x/(1+y+z), y/(1+x+z), z/(1+x+y))$

(i) $(1.0, 1.0, 1.0)$, $f(1.0, 1.0, 1.0) = \left(\frac{1.0}{1+1.0+1.0}, \frac{1.0}{1+1.0+1.0}, \frac{1.0}{1+1.0+1.0}\right) = (1/3, 1/3, 1/3)$

$= (.333, .333, .333)$

$f(.333, .333, .333) = \left(\frac{.333}{1+.333+.333}, \frac{.333}{1+.333+.333}, \frac{.333}{1+.333+.333}\right)$

$= \left(\frac{.333}{1.666}, \frac{.333}{1.666}, \frac{.333}{1.666}\right) = (.200, .200, .200)$

First 3 terms in Maple Notation are

[[1.0, 1.0, 1.0], [.333, .333, .333], [.200, .200, .200]]

(ii) Orb([x/(1+y+z), y/(1+x+z), z/(1+x+y)], [x,y,z], [1.0, 1.0, 1.0], 0, 2);

(iii) Orb([x/(1+y+z), y/(1+x+z), z/(1+x+y)], [x,y,z], [1.0, 1.0, 1.0], 1000, 1000)[1];

You get [0.0004997501157, 0.0004997501157, 0.0004997501157]

* There is no P10!

P11: $x(n) = x(n-1)^2 - 2x(n-1) + 2$

Underlying function $f(x) = x^2 - 2x + 2$

$$x = x^2 - 2x + 2$$
$$x^2 - 3x + 2 = 0$$
$$(x-1)(x-2) = 0$$
$$x = 1 \quad x = 2$$

Two equilibrium solutions $x(n) = 1$ & $x(n) = 2$

P12: $x(n) = 5/2\, x(n-1)(1 - x(n-1))$

Underlying Function $f(x) = 5/2\, x(1-x)$

$$x = 5/2\, x(1-x)$$
$$x = 5/2\, x - 5/2\, x^2 \rightarrow 5/2\, x^2 - 3/2\, x = 0$$
$$x(5/2\, x - 3/2) = 0$$
$$x = 0, \quad x = 3/5 \text{ or } .6$$

Two equilibrium solutions $x(n) = 0$, $x(n) = 3/5$ or $.6$

P13: $x(n) = k\,x(n-1)(1 - x(n-1))$

Underlying function $f(x) = kx(1-x)$

$$x = kx(1-x)$$
$$x = (kx - kx^2)$$
$$x - kx + kx^2 = 0$$
$$x((1-k) + kx) = 0$$
$$x = 0 \qquad kx = -(1-k) \rightarrow x = \frac{k-1}{k}$$

Two equilibrium solutions $x(n) = 0$, $x(n) = \frac{k-1}{k}$

P11': Find in attached Maple file

P12': Find in attached Maple file

P11'': $x(n) = x(n-1)^2 - 2x(n-1) + 2$

$f(x) = x^2 - 2x + 2$

$f'(x) = 2x - 2$

$f'(1) = 2(1) - 2 = 0 \rightarrow |0| < 1, \; x=1$ is stable

$f'(2) = 2(2) - 2 = 2 \rightarrow |2| > 1, \; x=2$ is unstable

P12'': $x(n) = \frac{5}{2}\, x(n-1)(1-x(n-1))$

$f(x) = \frac{5}{2} x(1-x) \rightarrow \frac{5}{2}x - \frac{5}{2}x^2$

$f'(x) = \frac{5}{2} - 5x$

$f'(0) = \frac{5}{2} - 5(0) = \frac{5}{2} \rightarrow |\frac{5}{2}| > 1 \quad x=0$ is unstable

$f'(3/5) = \frac{5}{2} - 5(3/5) = \frac{5}{2} - \frac{6}{2} = -\frac{1}{2} \rightarrow |-\frac{1}{2}| < 1$

$x = 3/5$ or $.6$ is stable

```
> read "/Users/jeton/Desktop/Math 336/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #P8 (iii.)
  Orb([1/(x+1)],[x],[0.5],1000,1000)[1];
```

[0.6180339887] **(2)**

```
> #P9 (iii.)
  Orb([x/(1+y+z),y/(1+x+z),z/(1+x+y)],[x,y,z],[1.0,1.0,1.0],1000,
  1000)[1];
```

[0.0004997501157, 0.0004997501157, 0.0004997501157] **(3)**

```
> #P11'

> Orb([x^2-2*x+2],[x],[1],1000,1010);
```

[[1], [1], [1], [1], [1], [1], [1], [1], [1], [1], [1]] **(4)**

```
> #Confirms that x(n) = 1 is indeed an equilibrium solution
```

```
> Orb([x^2-2*x+2],[x],[1.1],1000,1010);
```
[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000]] **(5)**

```
> Orb([x^2-2*x+2],[x],[.9],1000,1010);
```
[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000]] **(6)**

```
> #Starting close to x(n) = 1, the limit does still tend to 1 so we
  can say it is a stable equilibrium solution.
```

```
> Orb([x^2-2*x+2],[x],[2],1000,1010);
```
[[2], [2], [2], [2], [2], [2], [2], [2], [2], [2], [2]] **(7)**

```
> #Again, confirms x(n) = 2 is an equilibrium solution
```

```
> Orb([x^2-2*x+2],[x],[2.1],1000,1010);
```
[[Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)]] **(8)**

```
> Orb([x^2-2*x+2],[x],[1.9999],1000,1010);
```
[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000]] **(9)**

```
> #Even when we start around 2, the limit does not tend toward it,
  when we start a little over 2 we go to infinity, but a little under
  2 even at 1.999 we go back to 1. So x(n)=2 is not a stable
  equilibrium point.
```

```
> #P12'
```

```
> Orb([5/2*x*(1-x)],[x],[0],1000,1010);
```
[[0], [0], [0], [0], [0], [0], [0], [0], [0], [0], [0]] **(10)**

```
> #Confirms x(n) = 0 is an equilibrium solution
```

```
> Orb([5/2*x*(1-x)],[x],[0.001],1000,1010);
```
[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(11)**

```
> #Even when we start near 0 at .001 the limit goes towards .6 so x
  (n) = 0 is NOT a stable equilibrium solution.
```

```
> Orb([5/2*x*(1-x)],[x],[.6],1000,1010);
```
[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(12)**

```
> #Confirms x(n) = .6 is an equilibrium solution
```

```
> Orb([5/2*x*(1-x)],[x],[0.5],1000,1010);
```

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(13)**

```
> Orb([5/2*x*(1-x)],[x],[0.7],1000,1010);
```

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]] **(14)**

```
> #When we start around .6 the limit tends towards it so we can say
  that x(n) = .6 is a stable equilibrium solution.
```