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hw25, 12/2/2021

Questions on Pages 1→12

P1: Check whether $z=2$ is a solution of the equation $z^3+3z^2-11z+2=0$.
Is $z=3$ a solution

At $z=2$ ⇒ $2^3+3\cdot 2^2-11\cdot 2+2=8+12-22+2=0$ ✓
$z=2$ is a solution

At $z=3$ $3^3+3\cdot 3^2-11\cdot 3+2=27+27-33+2=23\neq 0$
$z=3$ is not a solution

P2: Check $z=\pi$ & $z=\frac{\pi}{2}$ as solutions to $\sin(z)=0$

$\sin(\pi)=0$, $z=\pi$ is a solution
$\sin(\pi/2)=1$, $z=\pi/2$ is not a solution

P3 Check $z=\frac{\pi}{3}$ as a solution $\sin^2(z)+\cos^2(z)=1$. Is $z=\frac{\pi}{5}$ a solution?

$\sin^2(z)+\cos^2(z)$, by the pythagorean identity, always equals 1.
Therefore, $\sin^2(z)+\cos^2(z)=1$ is a tautology.
By this logic, $z=\frac{\pi}{3}$ and $z=\frac{\pi}{5}$ are solutions for the given equation

P4 Find the set of all solutions for the trig equation $\sin^2(z) + \cos^2(z) = 1$

Based on the logic in P3

$z = \{x \mid x \in \mathbb{R}\}$, or the set of solutions is all real z-values.

P5 For the function $x(t) = t^4$, find the rate of change and the rate of change of the rate of change when $t=2$

$$x(t) = t^4$$

$$\frac{d}{dt} \Downarrow \quad x'(t) = 4t^3 \longrightarrow x'(2) = 32$$

$$\frac{d}{dt} \Downarrow \quad x''(t) = 12t^2 \longrightarrow x''(t) = 36$$

P6 Check whether $x=1$, $x=2$, $x=3$, $X=-1$, are fixed points of the function $f(x) = (x-1)(x-2)(x-3) + x$

$f(1) = 0\cdot(-1)\cdot(-2) + 1 = 1$ $\quad$ $x=1$ is a fixed point

$f(2) = (1)\cdot(0)\cdot(-1) + 2 = 2$ $\quad$ $x=2$ is a fixed point

$f(3) = (2)\cdot(1)\cdot(0) + 3 = 3$ $\quad$ $x=3$ is a fixed point

$f(-1) = (-2)(-3)(-4) - 1 = -25$, $\quad$ $x=-1$ is not a fixed point

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P7) Check the point $(0,-1) = (x,y)$ to be fixed for the function, and $(x,y) = (1,1)$

$f(x,y) = (x+y+1,\ x-y-2)$

$F(0,-1) = (0-1+1,\ 0+1-2) = (0,-1)$ $\quad$ $(x,y) = (0,-1)$ is a fixed point

$F(1,1) = (1+1+1,\ 1-1-2) = (3,-1)$ $\quad$ $(x,y) = (1,1)$ is not a fixed point

P8 For $f(x) = 1/x+1$

i. Find the first 3 terms of orbit starting at $x(0) = 0.5$

ii. Write the maple line that yields the same answer

iii. Use maple to find the 1000th term, what is it?

i- $f(0) = \underline{0.5}$ $f(1) = 1/(1.5) = \underline{2/3}$ $f(2) = 1/(5/3) = \underline{3/5}$

ii- Orb([1/(x+1)], [x], [0.5], 0, 2)

iii- Maple Command

Orb([1/(x+1)], [x], [0.5], 999, 1000)[2]; = $\underline{0.6180339887}$

P9 $f(x,y,z) = (x/(1+y+z), y/(1+x+z), z/(1+x+y))$

i. Find the first 3 terms of orbit starting at [1.0, 1.0, 1.0]

ii Write down the Maple Command that yields the same

iii Use maple to find 1000th term, what is it?

i- $f(0) = \underline{[1,1,1]}$ $f(1) = (1/1+1+1, 1/1+1+1, 1/1+1+1) = \underline{(1/3, 1/3, 1/3)}$

$f(2) = (1/3/(1+1/3+1/3), 1/3/(1+1/3+1/3), 1/3/(1+1/3+1/3)) = \underline{(1/5, 1/5, 1/5)}$

ii- Orb([x/(1+y+z), y/(1+x+z), z/(1+x+y)], [x,y,z], [1., 1., 1.], 0, 2)

0

iii- Orb([x/(1+y+z), y/(1+x+z), z/(1+x+y)], [x,y,z], [1., 1., 1.], 999,

1000][-1]; = $\underline{[0.000499, 0.000499, 0.000499]}$

No P10

P11 Find all equilibrium solutions of the given system

$$x(n) = \underbrace{x(n-1)^2 - 2\cdot x(n-1) + 2}_{f(x(n-1))}$$

Let $x = f(x) \to x = x^2 - 2x + 2 \to x^2 - 3x + 2 = 0 \to (x-2)(x-1) = 0$

Equilibrium solutions $x(n) = 2$ and $x(n) = 1$

P12 Find all equilibrium solutions for the system

$$x(n) = \underbrace{\frac{5}{2}\circ x(n-1)\circ(1 - x(n-1))}_{f(x(n-1))}$$

Let $x = f(x)$

$\to x = \frac{5}{2}\circ x\circ(1-x) \to 0 = \frac{3}{2}x - \frac{5}{2}x^2 \to 0 = x(3/2 - 5/2 x)$

Equilibrium Solutions $x(n) = 0$ and $x(n) = 3/5$

P13 Find all equilibrium solutions for the system

$$x(n) = \underbrace{K\circ x(n-1)\cdot(1 - x(n-1))}_{f(x(n-1))}$$

Let $x = f(x) \to x = K\circ x - K\cdot x^2 \to 0 = (K-1)x - K\circ x^2 \longrightarrow$

$\to 0 = x\cdot(K - 1 - K\circ x) \qquad x = 0, \; x = \frac{K-1}{K}$

Equilibrium Solutions are $x(n) = 0$ and $x(n) = \frac{K-1}{K}$

P11' and P12' Used the orb function at points relative to the found equilibrium points in 11&12 : in .txt file

P11': $x(n)=1$ is stable, $x(n)=2$ is not stable

P12': $x(n)=3/5$ is stable, $x(n)=0$ is not stable

P11" Use calculus to find the stable equilibrium solutions of

$$x(n) = \underbrace{x(n-1)^2 - 2\cdot x(n-1) + 2}_{f(x(n-1))}$$

$f'(x) = 2\cdot x - 2$

$f'(1) = 0$ $|0| < 1$ ✓ $x=1$ is a stable equilibrium solution

$f'(2) = 2$ $|2| < 1$ ✗ $x=2$ is an unstable equilibrium solution

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P12" Use calculus to find the stable equilibrium solutions of

$$x(n) = \underbrace{\frac{5}{2}\cdot x(n-1)\cdot(1-x(n-1))}_{f(x(n-1))}$$

$f'(x) = \frac{5}{2} - 5x$

$f'(0) = 5/2$ $|5/2| < 0$ ✗ $x=0$ is not a stable equilibrium solution

$f'(3/5) = \frac{5}{2} - 3 = -0.5$ $|-0.5| < 1$ ✓ $x=3/5$ is a stable equilibrium solution