```
> #OK to post Homework
  #Shreya Ghosh, 12-02-2021, Assignment 25
> read "/Users/shreyaghosh/Documents/DMB.txt"
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

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(1)

```
> #8iii)
> Orb([1/(x+1)], [x], [0.5], 1000, 1000)[1]
```

bad input

$FAIL_1$ **(2)**

```
> Orb([1/(x+1)], [x], [0.5], 1000, 1001)[1]
```

[0.6180339887] **(3)**

```
> #not outputting when entering the same numbers for K1 and K2
>
> #9iii)
> Orb([x/(1+y+z), y/(1+x+z), z/(1+x+y)], [x,y,z], [1.0,1.0,1.0], 1000, 1001)[1]
```

$$[0.0004997501157,\ 0.0004997501157,\ 0.0004997501157] \quad (4)$$

```
>
> #11')
> Orb([x^2 - 2·x + 2], [x], [2], 1000, 1010)
```

$$[[2],[2],[2],[2],[2],[2],[2],[2],[2],[2],[2],[2]] \quad (5)$$

```
> Orb([x^2 - 2·x + 2], [x], [2.1], 1000, 1010)
```

$$[[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}],[\text{Float(undefined)}]] \quad (6)$$

```
> Orb([x^2 - 2·x + 2], [x], [1], 1000, 1010)
```

$$[[1],[1],[1],[1],[1],[1],[1],[1],[1],[1],[1],[1]] \quad (7)$$

```
> Orb([x^2 - 2·x + 2], [x], [1.1], 1000, 1010)
```

$$[[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000],[1.000000000]] \quad (8)$$

```
> #x(n) = 2 is unstable, x(n) = 1 is stable
>
> #12')
> Orb([5/2·x·(1-x)], [x], [0], 1000, 1010)
```

$$[[0],[0],[0],[0],[0],[0],[0],[0],[0],[0],[0],[0]] \quad (9)$$

```
> Orb([5/2·x·(1-x)], [x], [0.1], 1000, 1010)
```

$$[[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000]] \quad (10)$$

```
> Orb([5/2·x·(1-x)], [x], [0.6], 1000, 1010)
```

$$[[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000]] \quad (11)$$

```
> Orb([5/2·x·(1-x)], [x], [0.7], 1000, 1010)
```

$$[[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000],[0.6000000000]] \quad (12)$$

```
[0.6000000000], [0.6000000000]]
> #x(n) = 0 is unstable, x(n)= 0.6 is stable
```

HW 25

1. $z^3 + 3z^2 - 11z + 2 = 0$

$2^3 + 3(2)^2 - 11(2) + 2 = 0$

$8 + 12 - 22 + 2 = 0$

$0 = 0$

2 is a solution

$z^3 + 3z^2 - 11z + 2 = 0$

$3^3 + 3(3)^2 - 11(3) + 2 = 0$

$27 + 27 - 33 + 2 = 0$

$23 \neq 0$

3 is not a solution

2. $\sin z = 0$

$\sin(\pi) = 0$

$0 = 0$

π is a solution

$\sin z = 0$

$\sin\left(\frac{\pi}{2}\right) = 0$

$1 \neq 0$

$\frac{\pi}{2}$ is not a solution

3. $\sin^2(z) + \cos^2(z) = 1$

$\sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) = 1$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

$\frac{3}{4} + \frac{1}{4} = 1$

$1 = 1$

$\frac{\pi}{3}$ is a solution

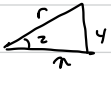
$\sin^2 z + \cos^2 z = 1$

$\sin^2\left(\frac{\pi}{5}\right) + \cos^2\left(\frac{\pi}{5}\right) = 1$

$1 = 1$

$\frac{\pi}{5}$ is a solution

4. $\sin^2 z + \cos^2 z = 1$



$x^2 + y^2 = r^2$

$\frac{x^2}{r^2} + \frac{y^2}{r^2} = \frac{r^2}{r^2} = 1 \Rightarrow$ set of solutions is the interval $[0, 2\pi]$

5. $x(t) = t^4$

$x'(t) = 4t^3 \Rightarrow x'(2) = 32$

$x''(t) = 12t^2 \Rightarrow x''(2) = 48$

6. $f(x) = (x-1)(x-2)(x-3) + x$

$1 = (1-1)(1-2)(1-3)+1$

$1 = 0 + 1$

$1 = 1 \Rightarrow$ 1 is a fixed point

$2 = (2-1)(2-2)(2-3)+2$

$2 = 0 + 2$

$2 = 2 \Rightarrow$ 2 is a fixed point

$3 = (3-1)(3-2)(3-3)+3$

$3 = 0 + 3$

$3 = 3 \Rightarrow$ 3 is a fixed point

$-1 = (-1-1)(-1-2)(-1-3)-1$

$-1 = (-2)(-3)(-4) - 1$

$-1 \neq -25 \Rightarrow$ -1 is not a fixed point

7. $f_x = x + y + 1$ $\quad f_y = x - y - 2$

$f_x(0,-1) = 0-1+1 = 0$ $\quad f_y(0,-1) = 0+1-2 = -1$

$(0,-1) = (0,-1) \Rightarrow$ (0,-1) is a fixed point

$f_x(1,1) = 1+1+1 = 3$ $\quad f_y(1,1) = 1-1-2 = -2$

$(1,1) \neq (3,-2) \Rightarrow$ (1,1) is not a fixed point

8. i) $f(x) = \frac{1}{x+1}$

$x(0) = 0.5$, $x(1) = \frac{1}{0.5+1} = 0.6667$, $x(2) = \frac{1}{0.6667+1} = 0.6$

ii) Orb([1/(x+1)], [x], [0.5], 0, 2);

iii) In Maple $\Rightarrow$ [0.6180339887]

9. i) $f(x,y,z) = \left(\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y}\right)$

$(1.0, 1.0, 1.0)$; $f(1.0, 1.0, 1.0) = (0.3333, 0.3333, 0.3333)$; $f(0.3333, 0.3333, 0.3333) = (0.2, 0.2, 0.2)$

ii) Orb([x/(1+y+z), y/(1+x+z), z/(1+x+y)], [x, y, z], [1.0, 1.0, 1.0], 0, 2);

iii) In Maple ⇒ [0.000499750115 7, 0.000499750115 7, 0.000499750115 7]

11 $x(n) = x(n-1)^2 - 2x(n-1) + 2$

$f(x) = x^2 - 2x + 2$

$x = x^2 - 2x + 2$

$0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

$x(n) = 2$, $x(n) = 1$

12. $x(n) = \frac{5}{2} x(n-1)(1 - x(n-1))$

$f(x) = \frac{5}{2} x(1-x)$

$x = \frac{5}{2} x(1-x)$

$x = \frac{5}{2} x - \frac{5}{2} x^2$

$\frac{5}{2} x^2 - \frac{3}{2} x = 0$

$5x^2 - 3x = 0$

$x(5x-3) = 0$

$x(n) = 0$, $x(n) = \frac{3}{5}$

13. $x(n) = kx(n-1)(1 - x(n-1))$

$f(x) = kx(1-x)$

$x = kx(1-x)$

$x = kx - kx^2$

$kx^2 + (1-k)x$

$x(kx + 1 - k) = 0$

$kx + 1 - k = 0$

$x = \frac{k-1}{k}$

$x(n) = 0$, $x(n) = \frac{k-1}{k}$

11.' In Maple

12.' In Maple

11". $x(n) = x(n-1)^2 - 2x(n-1) + 2$, $x(n) = 2$, $x(n) = 1$

$f(x) = x^2 - 2x + 2$

$f'(x) = 2x - 2$

$f'(2) = 2 \Rightarrow$ unstable

$f'(1) = 0 \Rightarrow$ stable

12". $x(n) = \frac{5}{2}x(n-1)\left(1 - x(n-1)\right)$, $x(n) = 0$, $x(n) = \frac{3}{5}$

$f(x) = \frac{5}{2}x - \frac{5}{2}x^2$

$f'(x) = \frac{5}{2} - 5x$

$f'(0) = \frac{5}{2} \Rightarrow$ unstable

$f'(\frac{3}{5}) = -0.5 \Rightarrow$ stable