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# Homework #25

P1. $z = 2$ solution of $z^3 + 3z^2 - 11z + 2 = 0$

$\Rightarrow \quad 2^3 + 3(2)^2 - 11(2) + 2 = 8 + 12 - 22 + 2 = 0$ ✓

$3^3 + 3(3)^2 - 11(3) + 2 = 27 + 27 - 33 + 2 \neq 0$

P2. $z = \pi, \frac{\pi}{2}$ solution of $\sin z = 0$

$\Rightarrow \quad \sin \pi = 0$ ✓

$\sin \frac{\pi}{2} = 1$ ×

P3. $z = \frac{\pi}{3}, \frac{\pi}{5}$ solution of $\sin^2 z + \cos^2 z = 1$

$\Rightarrow \quad \sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) = 1$ ✓

$\sin^2\left(\frac{\pi}{5}\right) + \cos^2\left(\frac{\pi}{5}\right) = 1$ ✓

P4. $\sin^2 z + \cos^2 z = 1$ solutions

$\Rightarrow$ All real numbers are solutions because this is a Pythagorean identity

P5. $x(t) = t^4$

$x'(t) = 4t^3 \quad x'(2) = 4(8) = 32$

$x''(t) = 12t^2 \quad x''(2) = 12(2)^2 = 48$

P6. $f(x) = (x-1)(x-2)(x-3) + x$

$f(1) = (1-1)(1-2)(1-3) + 1 = 0(-1)(-2) + 1 = 1$

$f(2) = (2-1)(2-2)(2-3) + 2 = 1(0)(-1) + 2 = 2$

$f(3) = (3-1)(3-2)(3-3) + 3 = 2(1)(0) + 3 = 3$

$f(-1) = (-1-1)(-1-2)(-1-3) - 1 = -2(-3)(-4) - 1 = -25$

$x = 1, 2, 3$ are fixed points

P7. $(x,y) = (0,-1)$ in $f(x,y) = (x+y+1, x-y-2)$

$\Rightarrow (0-1+1, 0+1-2) = (0,-1)$ is a fixed point

$(x;y) = (1,1)$ in $f(x,y) = (x+y+1, x-y-2)$

$\Rightarrow (1+1+1, 1-1-2) = (3,2)$

$\Rightarrow (1,1)$ is not a fixed point

P8. $f(x) = \dfrac{1}{x+1}$, $x(0) = 0.5$

$\Rightarrow x(0) = 0.5$

$x(1) = \dfrac{1}{0.5+1} = \dfrac{1}{1.5} = \dfrac{2}{3}$

$x(2) = \dfrac{1}{\frac{2}{3}+1} = \dfrac{1}{5/3} = \dfrac{3}{5}$

b) $\text{orb}([\frac{1}{(x+1)}], [x], [0.5], 0, 2);$

c) $\text{Orb}([\frac{1}{(x+1)}], [x], [0.5], 1000, 1000)[1];$

P9. $f(x,y,z) = \left(\dfrac{x}{1+y+z}, \dfrac{y}{1+x+z}, \dfrac{z}{1+x+y}\right)$

$\Rightarrow f(1,1,1) = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$

$f(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}) = (\frac{2}{10}, \frac{2}{10}, \frac{2}{10})$

$f(\frac{2}{10}, \frac{2}{10}, \frac{2}{10}) = (0.14285, 0.14285, 0.14285)$

P9. b) Orb([ $\frac{x}{1+y+z}$ , $\frac{y}{1+x+z}$ , $\frac{z}{1+x+y}$ ], [x,y,z], [1,1,1], 0, 2 ];

c) Orb([ $\frac{x}{1+y+z}$ , $\frac{y}{1+x+z}$ , $\frac{z}{1+y+y}$ ], [x,y,z], [1,1,1], 1000, 1000, [1] ;

P10. Not listed in worksheet

P11. $x(n) = x(n-1)^2 - 2x(n-1) + 2$

$\Rightarrow f(x) = x^2 - 2x + 2$

$\Rightarrow x = x^2 - 2x + 2$

$\Rightarrow x^2 - 3x + 2 = 0$

$\Rightarrow (x-2)(x-1) = 0$

$\Rightarrow$ $x(n)=2$, $x(n)=1$ are the equilibrium solutions

P12. $x(n) = \frac{5}{2} x(n-1)(1-x(n-1))$

$\Rightarrow f(x) = \frac{5}{2}x(1-x)$

$\Rightarrow x = \frac{5}{2}x(1-x)$

$\Rightarrow 0 = \frac{5}{2}x(1-x) - x$

$\Rightarrow 0 = \frac{5}{2}x - \frac{5}{2}x^2 - x$

$\Rightarrow 0 = x(\frac{5}{2} - \frac{5}{2}x - 1)$

$\Rightarrow 0 = x(\frac{3}{2} - \frac{5}{2}x)$

$\Rightarrow$ $x = 0$, $x = \frac{3}{5}$ are the equilibrium solutions

P13. $x(n) = kx(n-1)(1-x(n-1))$

$\Rightarrow f(x) = kx(1-x)$

$\Rightarrow x = kx - kx^2$

$\Rightarrow 0 = kx - x - kx^2$

$\Rightarrow 0 = (k-1)x - kx^2$

$\Rightarrow 0 = x((k-1) - kx)$

$\Rightarrow x = 0$, $x = \frac{k-1}{k}$ are the equilibrium solutions

P11'. $x(n) = x(n-1)^2 - 2x(n-1) + 2$

$\Rightarrow$ Orb$([x^2-2x+2], [x], [2], 1000, 1010)$

$\Rightarrow$ Orb$([x^2-2x+2], [x], [1], 1000, 1010)$

$x(n) = 2$ is unstable fixed point

$x(n) = 1$ is stable fixed point

P12' $x(n) = \frac{5}{2}x(n-1)(1-x(n-1))$

$\Rightarrow$ Orb$([\frac{5x}{2}(1-x)], [x], [0], 1000, 1010)$

$\Rightarrow$ Orb$([\frac{5x}{2}(1-x)), [x], [\frac{3}{5}], 1000, 1010)$

$\Rightarrow x(n) = \frac{3}{5}$ is stable fixed point

$x(n) = 0$ is unstable fixed point

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P11'' $x(n) = x(n-1)^2 - 2x(n-1) + 2$

$\Rightarrow f(x) = x^2 - 2x + 2$

$\Rightarrow f'(x) = 2x - 2$

$\Rightarrow f'(2) = 2$

$f'(1) = 0$

$\Rightarrow$ $x(n) = 2$ is unstable

$x(n) = 1$ is stable

P12'' $x(n) = \frac{5}{2} x(n-1)(1 - x(n-1))$

$\Rightarrow f(x) = \frac{5}{2} x(1-x) = \frac{5}{2}x - \frac{5}{2}x^2$

$\Rightarrow f'(x) = \frac{5}{2} - 5x$

$\Rightarrow f'(0) = \frac{5}{2} - 0 = \frac{5}{2}$

$f'(\frac{3}{5}) = \frac{5}{2} - 3 = \frac{1}{2}$

$\Rightarrow$ $x(n) = 0$ is unstable

$x(n) = \frac{3}{5}$ is stable

P14 $x'(t) = 2x(t)(1-x(t))(2-x(t))(3-x(t))$

$\Rightarrow f(x) = 2x(1-x)(2-x)(3-x) = 0$

$\Rightarrow$ $x = 0, 1, 2, 3$

iii) $f(x) = (2x - 2x^2)(2-x)(3-x)$

$\Rightarrow f(x) = (2x - 2x^2)(6 - 5x + x^2)$

$\Rightarrow f(x) = -2x^4 + 12x^3 - 22x^2 + 12x$

$\Rightarrow f'(x) = -8x^3 + 36x^2 - 44x + 12$

$\Rightarrow f'(0) = 12$

$f'(1) = -8 + 36 - 44 + 12 = -4$

$f'(2) = 4$

$f(3) = -12$

1 and 3 are sta