OK to post
Julian Herman
Assignment 25, 2nd December, 2021

1.) $z^3 + 3z^2 - 11z + 2 = 0$

$z = 2: \quad 2^3 + 3 \cdot 2^2 - 11(2) + 2 \stackrel{?}{=} 0$

$8 + 12 - 22 + 2$

$20 - 20 = 0$ ✓

$\boxed{z = 2 \text{ is a solution}}$

$z = 3: \quad 3^3 + 3 \cdot 3^2 - 11(3) + 2 \stackrel{?}{=} 0$

$27 + 27 - 33 + 2$

$54 - 31$

$23 \neq 0$

$\boxed{z = 3 \text{ is NOT a solution}}$

2.) $\sin(\pi) \stackrel{?}{=} 0$

$0 = 0$ ✓

$\boxed{z = \pi \text{ is a solution}}$

$\sin\left(\frac{\pi}{2}\right) \stackrel{?}{=} 0$

$1 \neq 0$

$\boxed{z = \frac{\pi}{2} \text{ is NOT a solution}}$

3.) $\sin^2(\pi/3) + \cos^2(\frac{\pi}{3}) \overset{?}{=} 1$

$\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$\frac{3}{4} + \frac{1}{4}$

$1 = 1$ ✓

$\boxed{z = \frac{\pi}{3} \text{ is a solution}}$

$\sin^2(\frac{\pi}{5}) + \cos^2(\frac{\pi}{5}) \overset{?}{=} 1$

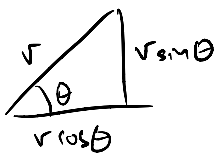
$0.3846 + 0.6545 \approx 1$ ✓

from rounding

$\boxed{z = \frac{\pi}{5} \text{ is a solution}}$

4.) The set of all solutions is the set of all real numbers:

$\boxed{z \in \mathbb{R}}$



From Pythagoreas: $a^2 + b^2 = c^2$

$r^2\cos^2\theta + r^2\sin^2\theta = r^2$

$r^2(\cos^2\theta + \sin^2\theta) = r^2$

$\cos^2\theta + \sin^2\theta = 1$ for any value $\theta \in \mathbb{R}$

5.) $x(t) = t^4$

rate of change: $x'(t) = 4t^3$

when $t = 2$: $x'(2) = 4 \cdot 2^3 = 32$

rate of change of the rate of change: $x''(t) = 12t^2$

when $t = 2$: $x''(2) = 12 \cdot 2^2 = 48$

6) For x to be a fixed point: $f(x) = x$

$$f(1) = \overset{0}{(1-1)}\cancel{(1-2)(1-3)} + 1$$

$$f(1) = 1 \quad \checkmark \quad \boxed{x=1 \text{ is a F.P.}}$$

$$f(2) = \cancel{(2-1)}\overset{0}{\cancel{(2-2)}}(2-3) + 2$$

$$f(2) = 2 \quad \checkmark \quad \boxed{x=2 \text{ is a F.P.}}$$

$$f(3) = \cancel{(3-1)(3-2)}\overset{0}{(3-3)} + 3$$

$$f(3) = 3 \quad \checkmark \quad \boxed{x=3 \text{ is a F.P.}}$$

$$f(-1) = (-1-1)(-1-2)(-1-3) - 1$$

$$(-2)\cdot(-3)(-4) - 1$$

$$6\cdot(-4) - 1$$

$$-24 - 1$$

$$-25$$

$$f(-1) = -25 \neq -1$$

$$\boxed{x=-1 \text{ is NOT a F.P.}}$$

7) $f(0,-1) = (0-1+1,\ 0+1-2) = (0,-1)$ $\checkmark$

$$\boxed{(0,-1) \text{ is a F.P.}}$$

$$f(1,1) = (1+1+1,\ 1-1-2) = (3,-2)$$

$$(1,1) \neq (3,-2)$$

$$\boxed{(1,1) \text{ is NOT a F.P.}}$$

8.) i) $x(0) = 0.5$

$$x(1) = \frac{1}{0.5+1} = \frac{2}{3} = 0.667$$

$$x(2) = \frac{1}{\frac{2}{3}+1} = \frac{1}{\frac{5}{3}} = \frac{3}{5} = 0.6$$

$$[[0.5],[0.667],[0.6]]$$

ii) $Orb([\frac{1}{x+1}],[x],[0.5],0,2);$

iii) $Orb([\frac{1}{x+1}],[x],[0.5],1000,1000)[1]$

$$[0.6180339887]$$

9.) i) 1st term: $[1.0, 1.0, 1.0]$

2nd term: $\left[\frac{1}{1+1+1}, \frac{1}{1+1+1}, \frac{1}{1+1+1}\right] = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$

$= [0.33, 0.33, 0.33]$

3rd term: $\left[\frac{1/3}{1+\frac{1}{3}+\frac{1}{3}}, \frac{1/3}{1+\frac{1}{3}+\frac{1}{3}}, \frac{1/3}{1+\frac{1}{3}+\frac{1}{3}}\right]$

$= [0.2, 0.2, 0.2]$

Maple notation :

$$[[1.0,1.0,1.0],[0.33,0.33,0.33],[0.2,0.2,0.2]]$$

ii) $Orb\left(\left[\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y}\right], [x,y,z], [1.0, 1.0, 1.0], 0, 2\right)$

iii) $Orb\left(\left[\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y}\right], [x,y,z], [1.0, 1.0, 1.0], 1000, 1000\right)[1]$

$[0.000499750157, 0.000499750157, 0.000499750157]$

11.) $x(n) = x(n-1)^2 - 2x(n-1) + 2$

Underlying function: $f(x) = x^2 - 2x + 2$

$f(x) = x$: $x = x^2 - 2x + 2$

$$x^2 - 3x + 2 = 0$$

$$(x-2)(x-1) = 0$$

$$x = 2, \quad x = 1$$

$\boxed{\text{Equilibrium solutions: } x(n) = 2, \ x(n) = 1}$

12.) $x(n) = \frac{5}{2} \cdot x(n-1)(1 - x(n-1))$

Underlying function: $f(x) = \frac{5}{2}x(1-x)$

$f(x) = x$: $x = \frac{5}{2}x(1-x)$

$$x - \frac{5}{2}x(1-x) = 0$$

$$x\left(1 - \frac{5}{2} + \frac{5}{2}x\right) = 0$$

$$x\left(-\frac{3}{2}+\frac{5}{2}x\right)=0$$

$$x=0, \quad \frac{5}{2}x=\frac{3}{2}$$

$$x=\frac{3}{5}$$

$\boxed{\text{EQ. solutions: } x(n)=0, \quad x(n)=\frac{3}{5}}$

13.) $x(n)=K\cdot x(n-1)\left(1-x(n-1)\right)$

Underlying function: $f(x)=Kx(1-x)$

$f(x)=x$: $x=Kx(1-x)$

$$x-Kx(1-x)=0$$

$$x(1-K+Kx)=0$$

$\boxed{x=0,}$ $\quad K-Kx=1$

$$K(1-x)=1$$

$$1-x=\frac{1}{K}$$

$$1=\frac{1}{K}+x$$

$$x=1-\frac{1}{K}$$

$$\boxed{x=\frac{K-1}{K}}$$

EQ solutions in terms of parameter K:

$x(n)=0$,

$x(n)=\frac{K-1}{K}$