

John Hermit

HW 25

$$\begin{aligned} P1. \quad z^3 + 3z^2 - 11z + 2 &= 0 \\ 2^3 + 3(2)^2 - 11(2) + 2 &= 0 \\ 8 + 12 - 22 + 2 &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} 3^3 + 3(3)^2 - 11(3) + 2 &= 0 \\ 27 + 27 - 33 + 2 &= 0 \\ 23 &\neq 0 \end{aligned}$$

$$\begin{aligned} P2. \quad \sin(z) &= 0 \\ \sin(\pi) &= 0 \\ 0 &= 0 \end{aligned}$$

$$\begin{aligned} \sin\left(\frac{\pi}{2}\right) &= 0 \\ 1 &\neq 0 \end{aligned}$$

$$\begin{aligned} P3. \quad \sin^2(z) + \cos^2(z) &= 1 \\ \sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) &= 1 \\ \frac{3}{4} + \frac{1}{4} &= 1 \\ 1 &= 1 \end{aligned}$$

$$\begin{aligned} \sin^2\left(\frac{\pi}{5}\right) + \cos^2\left(\frac{\pi}{5}\right) &= 1 \\ \frac{5 - \sqrt{5}}{8} + \frac{(1 + \sqrt{5})^2}{16} &= 1 \\ 1 &= 1 \end{aligned}$$

$$\begin{aligned} P4. \quad \sin^2 z + \cos^2 z &= 1 \\ z &= \mathbb{R} \end{aligned}$$

$$P5. \quad x(x) = x^4$$

$$x'(x) = 4x^3$$

$$x'(2) = 4(2)^3$$

$$x'(2) = 32$$

$$x''(x) = 12x^2$$

$$x''(2) = 12(2)^2$$

$$x''(2) = 48$$

$$P6. \quad f(x) = (x-1)(x-2)(x-3) + x$$

$$1 = (1-1)(1-2)(1-3) + 1$$

$$1 = 1$$

$$2 = (2-1)(2-2)(2-3) + 2$$

$$2 = 2$$

$$3 = (3-1)(3-2)(3-3) + 3$$

$$3 = 3$$

$$-1 = (-1-1)(-1-2)(-1-3) - 1$$

$$-1 \neq -25$$

$$P7. \quad f(x, y) = (x+y+1, x-y-2)$$

$$(0, -1) = (0-1+1, 0+1-2)$$

$$(0, -1) = (0, -1)$$

$$(1, 1) = (1+1+1, 1-1-2)$$

$$(1, 1) \neq (3, -2)$$

P8. $f(x) = \frac{1}{x+1}$

i. $x(0) = 0.5$

$$x(1) = \frac{1}{0.5+1} = \frac{2}{3}$$

$$x(2) = \frac{1}{\frac{2}{3}+1} = \frac{3}{5}$$

ii. $\text{Orb}([1/(x+1)], [x], [0.5], 0, 2)$,

iii. $\text{Orb}([1/(x+1)], [x], [0.5], 1000, 1000)[1]$,

you get 0.6180339887

P9. $f(x, y, z) = \left(\frac{x}{1+x+z}, \frac{y}{1+x+z}, \frac{z}{1+x+z} \right)$

i. $x(0) = [1, 1, 1]$

$$x(1) = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right]$$

$$x(2) = \left[\frac{9}{5}, \frac{9}{5}, \frac{9}{5} \right]$$

ii. $\text{Orb}([x/(1+x+z), y/(1+x+z), z/(1+x+z)], [x, y, z], [1, 1, 1], 0, 2)$,

iii. $\text{Orb}([x/(1+x+z), y/(1+x+z), z/(1+x+z)], [x, y, z], [1, 1, 1], 1000, 1000)[1]$

you get $\left[\frac{1}{2001}, \frac{1}{2001}, \frac{1}{2001} \right]$

$$P11. x(n) = x(n-1)^2 - 2x(n-1) + 2$$

$$f(x) = x^2 - 2x + 2$$

$$x = x^2 - 2x + 2$$

$$0 = x^2 - 3x + 2$$

$$x = 3, -1$$

$$P12. x(n) = \frac{5}{2} x(n-1) (1 - x(n-1))$$

$$x = \frac{5}{2} x (1 - x)$$

$$x = \frac{5}{2} x - \frac{5}{2} x^2$$

$$0 = -\frac{5}{2} x^2 + \frac{3}{2} x$$

$$0 = -5x^2 + 3x$$

$$0 = x(-5x + 3)$$

$$x = 0, \frac{3}{5}$$

$$P13. x(n) = K x(n-1) (1 - x(n-1))$$

$$x = K x (1 - x)$$

$$x = Kx - Kx^2$$

$$0 = (K-1)x - Kx^2$$

$$0 = x((K-1) - Kx)$$

$$x = 0, \frac{K-1}{K}$$

$$P11'' \quad x(n) = x(n-1)^2 - 2x(n-1) + 2$$

$$f(x) = x^2 - 2x + 2$$

$$f'(x) = 2x - 2$$

$$f'(3) = 4 \quad \text{unstable}$$

$$f'(-1) = -4 \quad \text{unstable}$$

$$P12'' \quad x(n) = \frac{5}{2} \cdot x(n-1) (1 - x(n-1))$$

$$f(x) = \frac{5}{2} x (1 - x)$$

$$f(x) = \frac{5}{2} x - \frac{5}{2} x^2$$

$$f'(x) = \frac{5}{2} - 5x$$

$$f'(0) = \frac{5}{2} \quad \text{unstable}$$

$$f'\left(\frac{3}{5}\right) = \frac{5}{2} - 3$$

$$f'\left(\frac{3}{5}\right) = -\frac{1}{2} \quad \text{stable}$$

> #John Hermitt hw25
read "/John/Rutgers/Senior Fall/Dynamic Models/DMB.txt" :
First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous) accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

*The most current version is available on WWW at:
<http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt> .
Please report all bugs to: DoronZeil at gmail dot com .*

*For general help, and a list of the MAIN functions,
type "Help()". For specific help type "Help(procedure_name);"*

*For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);*

*For a list of the functions that give examples of Discrete-time dynamical systems (some famous),
type: HelpDDM());*

For help with any of them type: Help(ProcedureName);

*For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM());
For help with any of them type: Help(ProcedureName);*

(1)

> #P8

$$\text{Orb}\left(\left[\frac{1}{x+1}\right], [x], [0.5], 999, 1000\right);$$
$$[[0.6180339887], [0.6180339887], [0.6180339887]]$$

(2)

> #P9

$$\text{Orb}\left(\left[\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y}\right], [x, y, z], [1, 1, 1], 1000, 1001\right)[1];$$
$$\left[\frac{1}{2001}, \frac{1}{2001}, \frac{1}{2001}\right]$$

(3)

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> #P11'
  OrbF([x^2 - 2·x + 2], [x], [2.99], 1000, 1010);
  OrbF([x^2 - 2·x + 2], [x], [-0.99], 1000, 1010)
[[Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [
  Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [
  Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)]]
[[Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [
  Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [
  Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)]]

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> #P12'
  #x=0 is unstable and x=0.6 is stable
  OrbF([ [ 5/2 · x · (1 - x) ], [x], [0.01], 1000, 1010 );
  OrbF([ [ 5/2 · x · (1 - x) ], [x], [0.5], 1000, 1010 );

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],
  [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],
  [0.6000000000], [0.6000000000]]
[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],
  [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],
  [0.6000000000], [0.6000000000]]

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(5)

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