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HW 25

P1: $z^3 + 3z^2 - 11z + 2 = 0$

$$(2)^3 + 3(2)^2 - 11(2) + 2 = 0$$

$$8 + 12 - 22 + 2 = 0$$

$$0 = 0 \quad \checkmark$$

$z=2$ is a solution

$$(3)^3 + 3(3)^2 - 11(3) + 2 = 0$$

$$27 + 27 - 33 + 2 = 0$$

$$54 - 33 + 2 = 0$$

$$23 \neq 0$$

$z=3$ is NOT a solution

P2: $\sin z = 0$

$$\sin(\pi) = 0$$

$$0 = 0 \quad \checkmark$$

$z = \pi$ is a solution

$$\sin\left(\frac{\pi}{2}\right) = 0$$

$$1 \neq 0$$

$z = \frac{\pi}{2}$ is NOT a solution

P3: $\left(\sin\left(\frac{\pi}{5}\right)\right)^2 + \left(\cos\left(\frac{\pi}{5}\right)\right)^2 = 1$

$$\left(\sin\left(\frac{\pi}{5}\right)\right)^2 + \left(\cos\left(\frac{\pi}{5}\right)\right)^2 = 1$$

$$1 = 1 \quad \checkmark$$

$z = \frac{\pi}{5}$ and $z = \frac{\pi}{5}$ are solutions

P4: $z = [0, \infty)$ is the set of all solutions

P5:

$$z(t) = t^4$$

$$z'(t) = 4t^3$$

$$z''(t) = 12t^2$$

$$z'(2) = 32$$

$$z''(2) = 48$$

P6:

$$f(1) = (1-1)(1-2)(1-3) + 1 = 1$$

$$f(2) = (2-1)(2-2)(2-3) + 2 = 2$$

$$f(3) = (3-1)(3-2)(3-3) + 3 = 3$$

$z=1, z=2, z=3$ are
fixed points

$$f(-1) = (-1-1)(-1-2)(-1-3) - 1 = -25$$

$z=-1$ is NOT a fixed point

P7:

$$f(0, -1) = (0-1+1, 0+1-2) = (0, -1)$$

$(x, y) = (0, -1)$ is a fixed point

$$f(1, 1) = (1+1+1, 1-1-2) = (3, -2)$$

$(x, y) = (1, 1)$ is NOT a fixed point

P8:

$$z(n) = \frac{1}{z(n-1) + 1}$$

$$(i) \quad z(0) = \boxed{0.5} \quad z(1) = \frac{1}{0.5+1} = \frac{2}{3} = \boxed{0.67} \quad z(2) = \frac{1}{\frac{2}{3}+1} = \frac{3}{5} = \boxed{0.6}$$

$$(ii) \quad \text{Orb}([1/(z+1)], [z], [0.5], [0, 2]);$$

$$(iii) \quad \text{Orb}([1/(z+1)], [z], [0.5], [1000, 1000]) [1];$$

$$z(1000) = 0.6180339887$$

P9:

$$(i) (1, 0, 1, 0) \quad f(1, 0, 1, 0) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) \quad f\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$$

$$(ii) \text{Ob}([z/(1+y+z), y/(1+z+z), z/(1+x+y)], [x, y, z], [1, 0, 1, 0], 0, 2);$$

$$(iii) \text{Ob}([z/(1+y+z), y/(1+z+z), z/(1+x+y)], [x, y, z], [1, 0, 1, 0], 1000, 1000)[1];$$
$$x(1000) = \left(\frac{1}{2001}, \frac{1}{2001}, \frac{1}{2001}\right)$$

P11: $z(n) = f(z(n-1))$

$$f(x) = x^2 - 2x + 2$$

$$x = x^2 - 2x + 2$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$z(n) = 1 \quad \text{and} \quad z(n) = 2$$

P12: $f(x) = \frac{5}{2}x(1-x)$

$$x = \frac{5}{2}x(1-x)$$

$$2x = 5x - 5x^2$$

$$5x^2 - 3x = 0$$

$$x(5x-3) = 0$$

$$z(n) = 0 \quad \text{and} \quad z(n) = \frac{3}{5}$$

P13: $f(x) = Kx(1-x)$

$$x = Kx(1-x)$$

$$x = Kx - Kx^2$$

$$Kx^2 + x - Kx = 0$$

$$Kx^2 + x(1-K) = 0$$

$$x[Kx + (1-K)] = 0$$

$$z(n) = 0 \quad \text{and} \quad z(n) = \frac{-(1-K)}{K}$$

$$P11' : \text{Orb}([z^2 - 2z + 2], [z], [1.01], 1000, 1010);$$

$$\text{Orb}(\text{~~~~~} [0.99] \text{~~~~~});$$

$$\text{Orb}(\text{~~~~~} [2.01] \text{~~~~~});$$

$$\text{Orb}(\text{~~~~~} [1.99] \text{~~~~~});$$

$x(n) = 1$ is stable $x(n) = 2$ is unstable
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$$P12' : \text{Orb}([\frac{5}{2}x(1-x)], [x], [0.01], 1000, 1010);$$

$$\text{Orb}(\text{~~~~~} [-0.01] \text{~~~~~});$$

$$\text{Orb}(\text{~~~~~} [0.59] \text{~~~~~});$$

$$\text{Orb}(\text{~~~~~} [0.61] \text{~~~~~});$$

$x(n) = 0$ is unstable $x(n) = 0.6$ is stable
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$$P11'' : f(z) = z^2 - 2z + 2$$

$$f'(z) = 2z - 2$$

$$f'(1) = 2 - 2 = 0 < 1$$

$$f'(2) = 4 - 2 = 2 > 1$$

$x(n) = 1$

$$P12'' : f(z) = \frac{5}{2}x(1-x)$$

$$f(z) = \frac{5}{2}z - \frac{5}{2}z^2$$

$$f'(z) = \frac{5}{2} - 5z$$

$$f'(0) = \frac{5}{2} > 1$$

$$f'(\frac{2}{5}) = \frac{5}{2} - 5 = -\frac{5}{2} < 1$$

$x(n) = 0.6$
