```
> #Charles Griebel Homework 25
  #OK to post
> read `C:/Users/cgrie/Dynam Models Bio/Homeworks/HW24/DMB.txt`
```

First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)
accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

The most current version is available on WWW at:
http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt .
Please report all bugs to: DoronZeil at gmail dot com .

For general help, and a list of the MAIN functions,
type "Help();". For specific help type "Help(procedure_name);"

For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);

For a list of the functions that give examples of Discrete-time dynamical systems (some famous), type: HelpDDM();
For help with any of them type: Help(ProcedureName);

For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM();
For help with any of them type: Help(ProcedureName);

------------------------------ **(1)**

```
> #P8:
  #(ii)
  print(`first 3 terms of orbit`);
  Orb([1/(x+1)],[x],[0.5],0,2);
  #(iii)
  print(`1000th term`);
  print(`because Orb() starts at 0, 999 is the index instead of
  1000`);
  Orb([1/(x+1)],[x],[0.5],999,999)[1][1];
```

first 3 terms of orbit

$$[[0.5], [0.6666666667], [0.5999999999]]$$

1000th term

because Orb() starts at 0, 999 is the index instead of 1000

$$0.6180339887 \tag{2}$$

```
> #P9
  #(ii)
  Orb([x/(1+y+z),y/(1+x+z),z/(1+x+y)],[x,y,z],[1.0,1.0,1.0],0,2);
  #(iii)
  print(`1000th term`);
  Orb([x/(1+y+z),y/(1+x+z),z/(1+x+y)],[x,y,z],[1.0,1.0,1.0],999,
  999)[1][1];
```

[[1.0, 1.0, 1.0], [0.3333333333, 0.3333333333, 0.3333333333], [0.2000000001, 0.2000000001, 0.2000000001]]

1000th term

$$0.0005002501157 \tag{3}$$

```
> #P12
  FP([(5/2)*x*(1-x)],[x]);
```

$$\left\{[0], \left[\frac{3}{5}\right]\right\} \tag{4}$$

```
> #P11'
  utRHS := [x^2-2*x+2];
  print(`test Equilibrium x(n)=1 by starting at x(0)=1.9`);
  Orb(utRHS,[x],[1.9],1000,1010);
  print(`test Equilibrium x(n)=2 by starting at x(0)=2.1`);
  Orb(utRHS,[x],[2.1],1000,1010);
  print(`at a high term (1001th-1011th digit, the trajectories
  indicate x(n)=1 is the only stable equilibrium solution`);
```

$$utRHS := [x^2 - 2x + 2]$$

test Equilibrium x(n)=1 by starting at x(0)=1.9

[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000]]

test Equilibrium x(n)=2 by starting at x(0)=2.1

[[Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)]]

at a high term (1001th-1011th digit, the trajectories indicate x(n)=1 is the only stable equilibrium solution **(5)**

```
> #P12"'
  #Using numerics, find all the stable equilibrium solutions.
  utRHS := [(5/2)*x*(1-x)];
  print(`for a starting point close to 3/5, we see x(n)=3/5 is
```

```
stable equilibrium solution`);
Orb(utRHS,[x],[0.61],1000,1010);
print(`for a starting point close to 0, we see x(n)=0 is not a
stable equilibrium solution`);
Orb(utRHS,[x],[-0.01],1000,1010);
```

$$utRHS := \left[\frac{5\,x\,(1-x)}{2} \right]$$

for a starting point close to 3/5, we see x(n)=3/5 is stable equilibrium solution

[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000]]

for a starting point close to 0, we see x(n)=0 is not a stable equilibrium solution

[[Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)], [Float(- ∞)]] **(6)**

Dynam Models bio HW 25

P1: Check whether $z = 2$ is a solution of the equation

$$z^3 + 3z^2 - 11z + 2 = 0$$

Test: $2(2)^3 + 3(2)^2 - 11(2) + 2 = 0$

$$8 + 12 - 22 + 2 = 0$$

$20 + 2 - 22 = 0 \Rightarrow z = 2$ is a solution.

For $z = 3$, we have

$$(3)^3 + 3(3)^2 - 33 + 2$$

$$= 81 + 27 - 33 + 2$$

$$= 83 - 33 + 27$$

$$= 50 + 27$$

$= 77$ which implies $z = 3$ is not a solution of

$$z^3 + 3z^2 - 11z + 2 = 0$$

Dynam Models Bio HW 25

P2: Check whether $z = \pi$ is a solution of the equation $\sin(z) = 0$

$\sin(\pi) = 0$ is correct b/c $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$

and ~~an~~ angle of π radians indicates that the length of the side opposite to the angle is 0.

is $z = \frac{\pi}{2}$ a solution to $\sin(\pi) = 0$

NO! $\sin(\frac{\pi}{2}) = 1 \neq 0$

P3: check whether $z = \frac{\pi}{3}$ is a solution to

$\sin^2(z) + \cos^2(z) = \underline{1}$

yes, $z = \frac{\pi}{3}$ is a solution b/c of pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = \underline{1}$

$z = \frac{\pi}{5}$ is also a solution for the same reason

P4: The set of all solutions of the equation

$\sin^2(z) + \cos^2(z) = \underline{1}$ Solution set: $z = \mathbb{R}$

Dynam Models Bio REVIEW
HW 25

P5: For the function $x(t) = t^4$, find its rate of change, and the rate of change of rate of change when time $t=2$

Rate of change at $t=2$ $= \frac{d}{dt} t^4 \Big|_{t=2}$

$= 4t^3 \Big|_{t=2}$

$= 24$

Rate of change of rate of change at $t=2$ $= \frac{d}{dt} 4t^3 \Big|_{t=2}$

$= 12t^2 \Big|_{t=2}$

$= 24$

P6. Check Whether $x=1$, $x=2$, $x=3$, $x=-1$ are the fixed points of the function

$f(x) = (x-1)(x-2)(x-3) + x$, $x=-1$

for $x=1$: $1 = 0 + 1 \Rightarrow x=1$ is a fixed point

$x=2$: $2 = 0 + 2 \Rightarrow x=2$ is a fixed point

$x=3$: $3 = 0+3 \Rightarrow x=3$ is a fixed point

$x=-1$: $-1 \neq -2(-3)(-4) + (-1) = -25$

WHICH MEANS

$x=-1$ is not a fixed point for f

Dynam Models Bio Review, HW 25

P7: Check whether the Point $(x,y) = (0,-1)$ is a fixed point of the transformation $f: \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$f(x,y) = (x+y+1,\ x-y-2).$$

Substituting $x=0$ and $y=-1$ into the above transformation yields

$$f(0,-1) = (0+(-1)+1,\ 0-(-1)-2)$$
$$= (0,\ -1)$$

Which implies $(0,-1) = (x,y)$ is a fixed point of f.

Is $(x,y) = (1,1)$ also a fixed point?

$$f(1,1) = (1+1+1,\ 1-1-2)$$
$$= (3,\ -2).$$

Because

$(x,y) \neq f(x,y)$ when $x=1$ and $y=1$, $(x,y) = (1,1)$ is not a fixed point.

Dynamic Models Bio HW 25

P8: For the function $f = \frac{1}{x+1}$

(i) By hand, find first 3 terms of orbit starting at $x(0) = 0.5$

$x(0) = 0.5$, $x(1) = \frac{1}{0.5+1} = \frac{2}{3}$

$x(2) = \frac{1}{\left(\frac{2}{3}\right)+1} = \frac{1}{\frac{2}{3}+\frac{3}{3}} = \frac{1}{\left(\frac{5}{3}\right)} = \frac{3}{5}$

(ii) Write down the maple line to get the same answer

Orb([1/(x+1)], [x], [0.5], 0, 2)

(iii) Using maple write the orb command to find the 100th term of the orbit.

What is it?

0.6180339887

Dynamic Models Bio HW 25

P9: For the transformation
$f(x,y,z) = (x/(1+y+z), y/(1+x+z), z/(1+x+y))$

By hand, find the first three terms of the orbit starting at [1.0, 1.0, 1.0]

First term: [1.0, 1.0, 1.0]

Second term:

$$f(1.0, 1.0, 1.0) = \left[\frac{1.0}{3.0}, \frac{1.0}{3.0}, \frac{1.0}{3.0}\right]$$

Third term:

$$f\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) = \left[\frac{\left(\frac{1}{3}\right)}{1+\frac{1}{3}+\frac{1}{3}}, \frac{\left(\frac{1}{3}\right)}{1+\frac{1}{3}+\frac{1}{3}}, \frac{\left(\frac{1}{3}\right)}{1+\frac{1}{3}+\frac{1}{3}}\right]$$

$$= \left[\frac{1}{3\left(\frac{5}{3}\right)}, \frac{1}{3\left(\frac{5}{3}\right)}, \frac{1}{3\left(\frac{5}{3}\right)}\right]$$

$$= \left[\frac{1.0}{5.0}, \frac{1.0}{5.0}, \frac{1.0}{5.0}\right]$$

25 Dynam Models Bio HW 25

P 11: Find all the equilibrium solutions of the first-order discrete time dynamical system

$x(n) = x(n-1)^2 - 2x(n-1) + 2$

Solution: The underlying transformation is:

$f(x) = x^2 - 2x + 2$

To get the fixed points of the transformation, Find the solutions to

$f(x) = x$

As follows:

Let $x = x^2 - 2x + 2$

which is a quadratic equation with factored form

$(x-2)(x-1) = 0 \Rightarrow x = 1$ and $x = 2$ are both fixed points

Therefore, $x=1$ and $x=2$ being fixed points of the underlying transformation implies

$x(n) = 1$ is an equilibrium solution

$x(n) = 2$ is an equilibrium solution

Dynami Models Bio HW 25

P12: Find all the equilibrium solutions of the first-order discrete time dependent system

$x(n) = \frac{5}{2} x(n-1)(1 - x(n-1))$

Equilibrium solutions are

$x(n) = 0$ and $x(n) = \frac{3}{5}$

are equilibrium solutions for all

P13 More generally, find all equilibrium solutions for

$x(n) = kx(n-1)(1 - x(n-1))$

When $f(x) = x$,

$$x = kx(1-x) \Leftrightarrow \frac{1}{k} = 1-x$$
$$\Leftrightarrow x = 1 - \frac{1}{k}$$

$\Rightarrow x(n) = 0$ is an equilibrium solution for all k

and

$x(n) = 1 - \frac{1}{k}$ is

also an equilibrium solution for all n

Dynam Models Bio HW 25

P11': Using Numerics, Find all stable equilibrium solutions of the first order dynamical system

$x(n) = x(n-1)^2 - 2x(n-1) + 2$

for each equilibrium solution found earlier (ANS ON Maple sheet)

P11'' USING CALCULUS,

Find all stable equilibrium solutions of the first-order discrete time dynamical system

$x(n) = x(n-1)^2 - 2x(n-1) + 2$

Step 1: Because this is a discrete case, ~~the deriva~~

$|f'(c)| < 1$ indicates a stable equilibrium when $x(n) = c$ is our equilibrium and f is the underlying transformation.

$f'(c) = 2c - 2$.

When $c = 1$, $f'(1) = 0 \Rightarrow |f'(1)| < 1$
$\Rightarrow x(n) = 1$ is stable

When $C = 2$, $f'(1) = 2 \Rightarrow |f'(2)| > 1 \Rightarrow x(n) = 2$ is not stable

Dynam Models Bio HW 25

P12" USING CALCULUS, find all of the stable equilibrium solutions of:

$$x(n) = \frac{5}{2} x(n-1)(1 - x(n-1))$$

UT: $f(x) = \frac{5}{2} x(1-x)$

$$\Rightarrow f'(x) = \frac{5}{2}\left[(1-x) + (-x)\right]$$

$$= \frac{5}{2}\left[1-2x\right]$$

$$= \frac{5}{2} - 5x$$

When $x = \frac{3}{5}$,

$$f'\left(\tfrac{3}{5}\right) = \frac{5}{2} - 3 = -\frac{1}{2}$$

$\Rightarrow \left|f'\left(\tfrac{3}{5}\right)\right| < 1$ Because Discrete

$\Rightarrow x(n) = \frac{3}{5}$ is stable equilibrium

When $x = 0$, $f'(0) = \frac{5}{2}$

$$\Rightarrow |f'(0)| > 1$$

$\Rightarrow x(n) = 0$ is not a stable equilibrium