

#not okay to post

#Anusha Nagar, Homework 25, 12.01.2021

Assignment: pages 1-12 all problems

P1 $z^3 + 3z^2 - 11z + 2 = 0$

Is $z=2$ a solution?

$$2^3 + 3 \cdot 2^2 - 11 \cdot 2 + 2 = ?$$

$$8 + 12 - 22 + 2 = 0 \quad \checkmark$$

Is $z=3$ a solution?

$$3^3 + 3 \cdot 3^2 - 11 \cdot 3 + 2 = 23 \quad \times$$

$z=2$ is a solution, $z=3$ is not

P2 $\sin z = 0$

Is $z=\pi$ a solution?

$$\sin(\pi) = 0 \quad \checkmark$$

Is $z=\frac{\pi}{2}$ a solution?

$$\sin\left(\frac{\pi}{2}\right) = 1 \quad \times$$

$z=\pi$ is a solution, $z=\frac{\pi}{2}$ is not

P3 $\sin^2 z + \cos^2 z = 1$

Is $z=\frac{\pi}{3}$ a solution?

$$\sin\left(\frac{\pi}{3}\right)^2 + \cos\left(\frac{\pi}{3}\right)^2 =$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4} + \frac{1}{4} = 1 \quad \checkmark$$

Is $z=\frac{\pi}{5}$ a solution?

$$\sin\left(\frac{\pi}{5}\right)^2 + \cos\left(\frac{\pi}{5}\right)^2 =$$

$$\Rightarrow \text{know } \sin^2(x) + \cos^2(x) = 1$$

\hookrightarrow all x are solutions $\Rightarrow \frac{\pi}{3}$ is solution

$\frac{\pi}{3}$ & $\frac{\pi}{5}$ are solutions

P4 All z are solutions

In a unit circle, if

we apply Pythagorean theorem,

$$\text{See } \sin^2(z) + \cos^2(z) = 1^2$$

Solution: {all real numbers}

P5 $x(t) = t^4$

$$x'(t) = 4t^3$$

$$\Rightarrow x'(2) = 32$$

$$x''(t) = 12t^2$$

$$\Rightarrow x''(2) = 48$$

P6 $f(x) = (x-1)(x-2)(x-3) + x$

Fixed points are where $f(a) = a$

$$f(1) = 0 + 1 = 1 \quad \checkmark$$

$$f(-1) = (-2)(-3)(-4) - 1 = -25 \quad \times$$

$$f(2) = 0 + 2 = 2 \quad \checkmark$$

$x=1, 2, 3$ are fixed points.

$$f(3) = 0 + 3 = 3 \quad \checkmark$$

$x=-1$ is not a fixed point.

P7 $F(x, y) = (x+y+1, x-y-2)$

Is $(0, -1)$ a FP?

$F(0, -1) = (0, -1)$ ✓

Is $F(1, 1)$ a FP?

$F(1, 1) = (3, -2)$ ✗

$(0, -1)$ is a fixed point, $(1, 1)$ is not a fixed point

P8 $F(x) = \frac{1}{x+1}$

(i) $x(0) = 0.5$

$x(1) = \frac{1}{1+0.5} = \frac{2}{3}$

$x(2) = \frac{1}{\frac{2}{3}+1} = \frac{3}{5}$

(ii) Orb $(\{1/(x+1)\}, [x], [0.5], 1000, 1000) [1]$;

(iii) 0.6180

P9 $F(x, y, z) = \left(\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y} \right)$

(i) $(1, 1, 1) \rightarrow F(1, 1, 1) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3} \right) \rightarrow F\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right) = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5} \right)$

(ii) Orb $(\{x/(1+y+z), y/(1+x+z), z/(1+x+y)\}, [x, y, z], [1, 0, 1, 0], 1000, 1000) [1]$;

(iii) $(0.00049975, 0.00049975, 0.00049975)$

P10 none in document

P11 $x(n) = x(n-1)^2 - 2x(n-1) + 2$

$f(x) = x^2 - 2x + 2$

$x = x^2 - 2x + 2$

$0 = x^2 - 3x + 2$

$0 = (x-2)(x-1)$

$x(n) = 1, x(n) = 2$ are equilibrium solutions

P12 $x(n) = \frac{5}{2}x(n-1)(1-x(n-1))$

$f(x) = \frac{5}{2}x(1-x)$

$x = \frac{5}{2}x(1-x)$

$x = \frac{5}{2}x - \frac{5}{2}x^2$

$\frac{5}{2}x^2 - \frac{5}{2}x = 0$

$x = 0, \frac{3}{5}$

$x(n) = 0, x(n) = \frac{3}{5}$ are equilibrium solutions

P13 $x(n) = kx(n-1)(1-x)$

$f(x) = kx(1-x)$

$x = kx(1-x)$

$x = kx - kx^2$

$kx^2 + x(1-k) = 0$

$x(kx + (1-k)) = 0$

$x = 0, \frac{k-1}{k}$

$\frac{\frac{5}{2} + 1}{\frac{5}{2}}$

$x(n) = 0, x(n) = \frac{k-1}{k}$ are equilibrium solutions

P11' See maple code. 1 is stable, 2 is not

P12' 0 not stable, $\frac{3}{5}$ stable

$$\boxed{P11''} \quad x(n) = x(n-1)^2 - 2x(n-1) + 2$$

Eq. points: 1, 2

$$f(x) = x^2 - 2x + 2$$

$$f'(x) = 2x - 2$$

$$f'(1) = |0| < 1 \Rightarrow \text{stable}$$

$$f'(2) = |2| > 1 \Rightarrow \text{unstable}$$

$$\boxed{P12''} \quad x(n) = \frac{5}{2}x(n-1)(1-x(n-1))$$

$$f(x) = \frac{5}{2}x(1-x) = \frac{5}{2}x - \frac{5}{2}x^2$$

Eq. pts: 0, $\frac{3}{5}$

$$f'(x) = \frac{5}{2} - 5x$$

$$f'(0) = \left|\frac{5}{2}\right| > 1 \Rightarrow \text{unstable}$$

$$f'\left(\frac{3}{5}\right) = \left|-\frac{1}{2}\right| = \frac{1}{2} < 1 \Rightarrow \text{stable}$$

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> #Not okay to post
> #Anusha Nagar, Homework 25, 12.02.2021
>
> read "C://Users/an646/Documents/DMB.txt"
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First Written: Nov. 2021

This is DMB.txt, A Maple package to explore Dynamical models in Biology (both discrete and continuous)

accompanying the class Dynamical Models in Biology, Rutgers University. Taught by Dr. Z. (Doron Zeilbeger)

*The most current version is available on WWW at:
<http://sites.math.rutgers.edu/~zeilberg/tokhniot/DMB.txt> .*

Please report all bugs to: DoronZeil at gmail dot com .

*For general help, and a list of the MAIN functions,
type "Help()". For specific help type "Help(procedure_name);"*

*For a list of the supporting functions type: Help1();
For help with any of them type: Help(ProcedureName);*

*For a list of the functions that give examples of Discrete-time dynamical systems (some famous),
type: HelpDDM());*

For help with any of them type: Help(ProcedureName);

*For a list of the functions continuous-time dynamical systems (some famous) type: HelpCDM());
For help with any of them type: Help(ProcedureName);*

(1)

```
> #P8
> Orb( [ [ 1 / (x + 1) ], [x], [0.5], 1000, 1000 ] [1];
[0.6180339887]
```

(2)

```
> #P9
> Orb( [ [ x / (1 + y + z), y / (1 + x + z), z / (1 + x + y) ], [x, y, z], [1.0, 1.0, 1.0], 1000, 1000 ] [1];
```

[0.0004997501157, 0.0004997501157, 0.0004997501157] (3)

> #P11'

> Orb($[x^2 - 2 \cdot x + 2]$, [x], [1.0], 1000, 1010);
[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000]] (4)

> Orb($[x^2 - 2 \cdot x + 2]$, [x], [1.1], 1000, 1010);
[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000]] (5)

> Orb($[x^2 - 2 \cdot x + 2]$, [x], [0.9], 1000, 1010);
[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000]] (6)

> #1 seems stable

> Orb($[x^2 - 2 \cdot x + 2]$, [x], [2.0], 1000, 1010);
[[2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.], [2.]] (7)

> Orb($[x^2 - 2 \cdot x + 2]$, [x], [2.1], 1000, 1010);
[[Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [
Float(undefined)], [Float(undefined)], [Float(undefined)], [Float(undefined)], [
Float(undefined)], [Float(undefined)], [Float(undefined)]] (8)

> Orb($[x^2 - 2 \cdot x + 2]$, [x], [1.9], 1000, 1010);
[[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000], [1.000000000], [1.000000000], [1.000000000], [1.000000000],
[1.000000000]] (9)

> #2 is not stable

>
>

> #P12'

> Orb($\left[\frac{5}{2} \cdot x \cdot (1 - x)\right]$, [x], [0.0], 1000, 1010);
[[0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.], [0.]] (10)

> Orb($\left[\frac{5}{2} \cdot x \cdot (1 - x)\right]$, [x], [0.1], 1000, 1010);
[[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],
[0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],
[0.6000000000]] (11)

> Orb($\left[\frac{5}{2} \cdot x \cdot (1 - x)\right]$, [x], [0.2], 1000, 1010);

```
[ [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000]]
```

(12)

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> #0 does not seem to be stable
```

```
> Orb( $\left[ \frac{5}{2} \cdot x \cdot (1 - x) \right]$ , [x], [0.6], 1000, 1010);
```

```
[ [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000]]
```

(13)

```
> Orb( $\left[ \frac{5}{2} \cdot x \cdot (1 - x) \right]$ , [x], [0.5], 1000, 1010);
```

```
[ [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000]]
```

(14)

```
> Orb( $\left[ \frac{5}{2} \cdot x \cdot (1 - x) \right]$ , [x], [0.7], 1000, 1010);
```

```
[ [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000], [0.6000000000],  
  [0.6000000000]]
```

(15)

```
> #0.6 seems stable
```

```
>
```