HW25 - Alan Ho

pgs 1-12

P1) $z^3 + 3z^2 - 11z + 2 = 0$

$(2)^3 + 3(2)^2 - 11(2) + 2 = 8 + 12 - 22 + 2 = 0$ ✓ $\Rightarrow$ 2 is a solution

$(3)^3 + 3(3)^2 - 11(3) + 2 = 27 + 27 - 33 + 2 = 24 \neq 0$ ✗ $\Rightarrow$ 3 is NOT a solution

P2) $\sin z = 0$

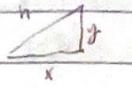
$\sin(\pi) = 0$ ✓ $\Rightarrow$ π is a solution $\quad \sin\left(\frac{\pi}{2}\right) = 1 \neq 0$ ✗ $\Rightarrow$ $\frac{\pi}{2}$ is NOT a solution

P3) $\sin^2 z + \cos^2 z = 1$

$\sin^2\left(\frac{\pi}{3}\right) + \cos^2\left(\frac{\pi}{3}\right) = 3/4 + 1/4 = 1$ ✓ $\Rightarrow$ $\pi/3$ is a solution

$\sin^2\left(\frac{\pi}{5}\right) + \cos^2\left(\frac{\pi}{5}\right) = 0.345 + 0.655 = 1$ ✓ $\Rightarrow$ $\frac{\pi}{5}$ is a solution

P4) $\sin^2 z + \cos^2 z = 1$



$\left(\frac{y}{h}\right)^2 + \left(\frac{x}{h}\right)^2 = \frac{y^2+x^2}{h^2} \Rightarrow$ by $x^2+y^2=h^2 = \frac{h^2}{h^2} \Rightarrow$ $\mathbb{R}$

- also true by identity $\sin^2 x + \cos^2 y = 1$

P5) $x(t) = t^4$ $\quad x'(t) = v(t) = 4t^3$, $\quad x''(t) = 12t^2 = a(t)$

$x(2) = 16$ $\quad v(2) = 32$ $\quad a(2) = 48$

P6) $f(x) = (x-1)(x-2)(x-3) + x$

$f(1) = 0 + 1 = 1$ ✓ $\Rightarrow$ 1 is a fixed point $\quad f(2) = 0 + 2 = 2$ ✓ $\Rightarrow$ 2 is fixed pt

$f(3) = 0 + 3 = 3$ ✓ $\Rightarrow$ 3 is fixed pt $\quad f(-1) = (-2)(-3)(-4) - 1 = -25 \neq -1$ ✗

$\Rightarrow$ -1 is NOT fixed pt

P7) $f(x,y) = (x+y+1, x-y-2)$

$f(0,-1) = (0, -\frac{1}{2}) \neq (0,-1)$ ✓ $\quad$ (0,-1) is NOT a fixed pt

$f(1,1) = (3,-2) \neq (1,1)$ ✗ $\quad$ (1,1) is NOT a fixed pt

P8) i) $f(y) = \frac{1}{x+1}$

$x(0) = 0.5 \quad x(1) = 2/3 \quad x(2) = 3/5$

ii) $\text{Orb}\left[\left[\frac{1}{x+1}\right], [x], [0.5], 0, 2\right]$

iii) $Orb\left[\left[\frac{1}{x+1}\right], [x], [0.4], 100, 1000\right)[1]$

$= \boxed{0.618}$

P9] i) $f(x,y,z) = \left(\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y}\right)$

$x(0) = [1,1,1]$ $x(1) = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ $x(2) = \left(\frac{3}{5}, \frac{3}{5}, \frac{3}{5}\right)$

$\left[\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right]$

ii) $Orb\left(\left[\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y}\right], [x,y,z], [1,1,1], 0, 2\right)$

iii) $Orb\left[\left[\frac{x}{1+y+z}, \frac{y}{1+x+z}, \frac{z}{1+x+y}\right], [x,y,z], [1,1,1], 100, 1000\right)[1]$

$= \boxed{\left[\frac{1}{5001}, \frac{1}{2001}, \frac{1}{2001}\right]}$

P11] $x(n) = x(n-1)^2 - 2x(n-1) + 2$

$f(x) = x^2 - 2x + 2 = x$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1)$ $\boxed{x = 1, 2}$

equil. solutions are $\boxed{x(n) = 1,\ x(n) = 2}$

P12] $x(n) = \frac{5}{2}x(n-1)\left(1 - x(n-1)\right)$

$f(x) = \frac{5}{2}x(1-x) = x$

$\frac{3}{2}x - \frac{5}{2}x^2 = 0 \Rightarrow \frac{3}{2}x\left(1 - \frac{5}{3}x\right) = 0$

$x = 0, \frac{3}{5}$

$\boxed{x(n) = 0,\ x(n) = \frac{3}{5}}$

P11'') $f'(x) = 2x - 2$

$f'(1) = 0 < 1 \Rightarrow \boxed{x = 1 \text{ is stable}}$

$f'(2) = 4 - 2 = 2 > 1 \Rightarrow \boxed{x = 2 \text{ is not stable}}$

P12'') $f'(x) = \frac{3}{2} - 5x$

$f'(0) = \frac{3}{2} > 1 \Rightarrow \boxed{x = 0 \text{ is not stable}}$

$f'\left(\frac{3}{5}\right) = \left|-\frac{3}{2}\right| > 1 \Rightarrow \boxed{x = \frac{3}{5} \text{ is not stable}}$