HW 23

1) For the following scenarios,

i) Decide whether it is discrete-time or continuous-time dynamical

ii) Set up the appropriate difference or differential equation

iii) Write explicitly the underlying transformation

iv) Find all the fixed points or equilibrium pts

v) Find all stable fixed points or all the stable equilibrium pts

a) The population of a certain species is decreasing at a rate that is twice its current value

b) The population of a certain species changes from one generation to the next. The value at a given generation is one half of its value at the previous generation.

c) The population of a certain species changes from one generation to the next. The value at a given generation is twice its value at the previous generation times

d) The population of a certain species scaled such that the maximum possible is 1 is increasing at a rate that is twice its current values times

a) i) In this system we have continuous time

ii) Twice its correct value, $\rightarrow x'(t) = -2x(t)$ ↑ derivative

iii) $f(x) = -2x \rightarrow$ function

iv) Solve for $-2x = 0 \Rightarrow x = 0$ i.e. $x = 0$ is only eq pt

v) $f'(x) = -2$, so $f'(0) = -2$, Thus we know this is stable

b) i) This system we have discrete time

ii) $x(n) = \frac{1}{2}x(n-1) \rightarrow$ one half

iii) $f(x) = \frac{1}{2}x$

iv) $f(x) = \frac{1}{2}x \Rightarrow x = \frac{1}{2}x \Rightarrow 0 = \frac{1}{2}x = \boxed{x = 0}$

v) $f'(x) = \frac{1}{2} \Rightarrow f'(0) = \frac{1}{2}$ thus this is stable

c) i) This system we have discrete time

ii) $x(n) = 2x(n-1)(1 - x(n-1))$

iii) $f(x) = 2x \cdot (1-x) \rightarrow$ removing $(n-1)$ underlying function

iv) $x = 2x \cdot (1-x) \Rightarrow 2x - (1-x) - x = 0$

Thus, we have $\{0, \frac{1}{2}\}$ as final pts.

v) $f'(x) = 2 - 4x$

$f'(0) = 2$ which is not stable

$f'(\frac{1}{2}) = 2 - 2 = 0$ which is stable

d) i) This system we have continuous time

ii) $x'(t) = 2x(t)(1 - x(t))$

iii) $f(x) = 2x(1-x)$ underlying function

iv) $2x - 2x^2 = [0, 1]$ eq pts

v) $f'(x) = 2 - 4x$

$f'(0) = 2$ which is not stable

$f'(1) = 2 - 4 = -2$ which is stable