Homework 23 *Ok to Post HW *Jeton Hida, Assignment 23, Nov 22, 2020

For this attendance quiz, I messed up and made dumb mistakes on the problems using continuous time because I was using separation of variables when it was uncalled for. Also, I made a mistake on (b), I knew I was supposed to test the value of the FP at derivative against 1, but I wrote $\frac{1}{2} > 1$ in haste and did not think twice about. For this I will do problems similar to a, b & d.

a1 The population of a current species is decreasing at a rate that is three times the current value.

Continuous

$x'(t) = -3x(t)$ $\quad f(x) = -3x$ $\quad -3x = 0$

Equ P $\rightarrow$ $x = 0$

$f'(x) = -3 \rightarrow f'(0) = -3 < 0$ set of stable Equ Points is $\{0\}$

a2 Population now decreases at a rate of half of the current value.

Continuous

$x'(t) = -\frac{1}{2}x(t)$ $\quad f(x) = -\frac{1}{2}x$ $\quad -\frac{1}{2}x = 0$

$x = 0$ only Equ P

$f'(x) = -\frac{1}{2}$ $\quad f'(0) = -\frac{1}{2} < 0$ set of stable Equ Points is $\{0\}$

b1 Population of a species changes from one generation to the next. The value at a given generation is equal to three-quarters of its value at the previous generation.

Discrete Time

$x(n) = \frac{3}{4}x(n-1)$ $\quad f(x) = \frac{3}{4}x$

$x = f(x) \rightarrow x = \frac{3}{4}x \rightarrow \frac{1}{4}x = 0$ $\quad x = 0$ is FP

$f'(x) = \frac{3}{4}$ so $f'(0) = \frac{3}{4} < 1$ absolute value less than 1

so the set of SFP $= \{0\}$

b2 The value at a given generation is now equal to five times the value at the previous generation

Discrete Time

$x(n) = 5x(n-1)$ $\quad f(x) = 5x$

$x = f(x) \rightarrow x = 5x \quad -4x = 0 \quad x = 0$ is FP

$f'(x) = 5 \rightarrow f'(0) = 5 > 1$ absolute value greater than one so this is not a stable FP at $x = 0$.

d1 The population of a certain species scaled such that the maximum possible is 1 is increasing at a rate that is four times its current value times (1 minus current value)

Continuous

$x'(t) = 4x(t)(1-x(t)) \quad f(x) = 4x(1-x)$

$4x(1-x) = 0 \rightarrow x = 0, 1$, set of EquP $= \{0, 1\}$

$f(x) = 4x - 4x^2 \rightarrow f'(x) = 4 - 8x \rightarrow f'(0) = 4 > 0$

this equilibrium is unstable

$f'(1) = 4 - 8 = -4 < 0$ this equilibrium is stable

Set of SEquP $= \{1\}$

d2 The population of a certain species scaled such that the maximum possible is 2 increases at rate that is six times the current values times (1 minus current value)

Continuous

$x'(t) = 6x(t)\left(1 - \frac{x(t)}{2}\right) \quad f(x) = 6x\left(1 - \frac{x}{2}\right) = 0$

$x = 0, \; x = 2$ is set of EquP $\{0, 2\}$

$f(x) = 6x - 3x^2 \rightarrow f'(x) = 6 - 6x \rightarrow f'(0) = 6 > 0$

so this equilibrium is unstable

$f'(2) = 6 - 12 = -6 < 0$ Set of SEquP $= \{2\}$