HW 22

1) $v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ $v_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ $v_3 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $v_4 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$ $v_5 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$

$Av_1 = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \end{pmatrix} = -1\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

v_1 is therefore an eigenvector and eigen value is -1

$Av_2 = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$

This not a multiple of v_2 thus is not an eigenvector

$Av_3 = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} = 2\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

Thus v_3 is an eiganvector and eigen value is 2

$Av_4 = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \end{pmatrix} = (-1)\begin{pmatrix} 3 \\ 6 \end{pmatrix}$

Thus v_4 is an eigen vector and eigen value is -1

$Av_5 = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Thus is not an eigen vector

2) The population of a certain bacteria increase at a rate that equals to a third of the reciprocal of the square of its current value. If at time $t=1$ its value is 1, what is the value at $t=8$?

Note: word problems are confusing and thus I struggle with them

Transform into a differential eq:

$x'(t) = \frac{1}{3x(t)^2}$ $\quad x(1) = 1$

Solve: $\frac{dx}{dt} = \frac{1}{3x^2}$

$\int 3x^2\, dx = \int dt$

$x^3 = t + C$

Then we use intial conditions

$z(1)=1$

so, $1^3 = 1 + C$

$C = 0$

Thus, $z^3 = t$

$z = \sqrt[3]{t}$

$\boxed{\text{at } t=8 \quad z=2}$

* 2) I am having trouble understanding and relating word problems to differential eq

I will have to review this in class

3) Conjecture 1: If $R_0 = a/(b+c) > 1$

then (x_n, y_n)

$\lim_{n \to \infty} (x_n, y_n) = (x^*, y^*)$

let $a = 4$

$b = 1 \quad c = 1$ then $\frac{4}{1+1} = 2 > 1$ ✓

so, (x_n, y_n)

1) $x_{n+1} = x_n(1-1-1) + y_n(1 - e^{-4x_n})$

2) $y_n = (1-y_n)1 + y_n e^{-4x_n}$

4) Conjecture 2: If $R_0 = a/b > 1$ then sol to (4)

$\lim x_n = x^*$

where $bx^* = (1-x^*)(1-e^{-ax^*})$

let $a = 2 \quad b = 1$

then $a/1 > 1$ ✓

so $1x^* = (1-x^*)(1-e^{-ax^*})$

HW 22

5) $$x(n) = \frac{x(n-1)}{10 + x(n-1)}$$

$x(0) \neq -10$

$f(x) = \frac{x}{10+x}$

so $0 = \frac{x}{10+x}$

then, $x = 0$ is only sol

then, $f'(x) = \frac{10}{(10-x)^2}$

$f'(0) = \frac{10}{100} = \boxed{1/10}$ thus stable